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互联非线性时滞系统的分散式自适应跟踪控制

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摘　要:为了实现系统对目标轨迹的快速准确跟踪,针对一类互联非线性时滞系统,提出了一种分散式自适应跟踪控制策略。通过使用极限学习机来处理系统中的未知非线性函数,引入Lyapunov-Krasovskii函数来处理未知时滞,结合反演控制技术和动态面控制技术,实现分散式自适应跟踪控制;基于Lyapunov稳定性理论以证明所设计的控制策略可以保证闭环系统跟踪误差一致且最终有界稳定,并借助两级化学反应釜系统验证所提控制策略的有效性。结果表明,所提控制策略能够有效处理系统中的非线性项与系统时滞,实现对目标轨迹的快速准确跟踪。所提策略能克服未知非线性和未知时滞对系统的影响,可为处理复杂非线性时滞系统提供参考。

关键词:自动控制理论;互联系统;输入饱和;时滞;反演控制;分散式跟踪控制

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Decentralized adaptive tracking control for interconnected nonlinear time delay systems

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Abstract: In order to achieve fast and accurate tracking of the target trajectory, a decentralized adaptive tracking control strategy was proposed for a class of interconnected nonlinear time-delay systems. By using the extreme learning machine to deal with the unknown nonlinear function in the system, the Lyapunov-Krasovskii function was introduced to deal with the unknown time delay. The backstepping control technique and the dynamic surface control technique were combined to realize decentralized adaptive tracking control. Based on Lyapunov stability theory, it is demonstrated that the designed control strategy can ensure consistent and ultimately bounded stability of the tracking errors of the closed-loop system, and the effectiveness of the proposed control strategy was verified by the two-stage chemical reactor system. The results show that the proposed control strategy can deal with the nonlinear term and time delay of the system effectively, and realize fast and accurate tracking of the target trajectory. The provided strategy overcomes the effect of unknown nonlinear and unknown time delay on

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the system, which provides an effective method for dealing with complex nonlinear time-delay systems.

Keywords: automatic control theory; interconnected systems; input saturation; timedelay; backstepping control; decentralized tracking control

近年来，关于非线性系统控制的研究一直是控制理论的研究热点，如战斗控制系统，化学过程控制和卫星系统控制[1-3]。由于现代系统的复杂性、非线性和日益增大的规模，单体系统已无法满足实际应用的需求。因此，由多个子系统耦合而成的互联非线性系统受到学者们的广泛关注。在实际应用中，时滞现象、系统动力学模型不确定是普遍存在，成为导致系统不稳定的重要因素。因此，针对互联非线性时滞系统的控制器设计及稳定性分析的研究具有重要意义。

在复杂的控制系统中，模糊逻辑系统和神经网络[4-10]因具有良好的逼近特性引起了学者们的高度关注。迄今为止，已经有许多优秀的研究成果。文献[11]基于 RBF 神经网络和自适应动态表面控制方法设计控制器，确保一类不确定随机非线性系统跟踪误差收敛到预定精度。文献[12]针对欠驱动自主水下机器人三维轨迹跟踪控制问题，提出 RBF 神经网络动态积分滑模控制器以实现对三维轨迹的跟踪。文献[13]针对 U 模型的非线性控制系统设计 Super-Twisting 控制器，保证了被控系统具有快速的跟踪性能和输出有界性。文献[14]针对一类具有周期扰动和输入时滞的不确定非线性系统，提出一种基于神经网络的自适应动态面控制策略，确保了闭环系统中所有信号是半全局有界的。文献[15]针对一类不确定严格反馈非线性系统，提出一种神经网络自适应跟踪控制算法。相较于神经网络，极限学习机具有更优越的控制性能[16]。极限学习机是一种单隐层前向网络的训练算法，特点是训练速度快，而且可以达到很高的泛化性能。极限学习机可以随机分配隐层节点的参数(包括输入权值和激活函数的偏差)，并且在训练过程中保持不变。到目前为止，极限学习机已经被广泛应用于识别和控制非线性动态系统当中[17-19]。文献[17]提出了基于在线序列极限学习机的模型预测控制，该方法的跟踪精度明显优于传统的基于神经网络的模型预测控制。文献[18]基于极限学习机神经网络算法及其主体结构的扩展，提出了一种新的在线误差最小化-极限学习机算法，不仅避免了网络冗余，而且大大提高了控制性能。针对双喷射发动机的空燃比控制问题，文献[19]提出了一种具有自适应控制器的极限学习机控制框架，其中极限学习机被用来识别基于引擎实时数据的最优控制率。综上所述，鉴于极限学习机优越的逼近性能，本文将借此来估计系统中的未知非线性项。

受被控制系统空间上大型化、结构上复杂化等因素的影响，传统的集中控制使得被控系统间的信息交流变得异常复杂，这不仅增加了系统的集成度，而且增加了系统的运行费用，同时降低了系统的可靠性。文献[20]针对一类具有非对称控制输入和不匹配互联项的非线性互联系统，提出一种分散学习控制策略，实现了闭环系统中所有信号一致最终有界。文献[21]针对具有非对称输入和不匹配互联项的连续时间非线性互联系统的分散镇定问题，设计了一种基于自适应评判的分散控制律，保证了被控系统的渐近稳定性。文献[22]提出一种分散自适应滑模控制方案，克服了系统存在死区线性输入和子系统间具有未知互联的问题。文献[23]提出一种全局分散非光滑跟踪算法，保证了一类具有附加扰动的强耦合互联系统的稳定。尽管上述工作已取得了一定的进展，但并没有考虑状态时滞给系统带来的影响。在许多实际应用中，系统的各部分间进行信息传输不可避免地会存在时延，这会给被控制系统带来不可预期的不稳定性。

基于上述分析，本文针对互联非线性时滞系统的自适应跟踪控制问题，通过使用极限学习机来估计系统中的非线性函数，利用 Lyapunov-Krasovskii 函数消除时滞对互联非线性系统的影响，基于反演控制技术和动态面控制技术提出分散式自适应跟踪控制策略，以使闭环系统中所有的信号最终一致有界。

1　系统模型和问题描述

考虑第 i 个互联非线性时滞子系统，其动力学模型如下：

$$\begin{cases}\dot{x}_{i,l}=x_{i,l+1}+f_{i,l}(\bar{x}_{i,l})+g_{i,l}(x_{i,l}(t-d_{i,l}))+\Delta_{i,l}(\bar{y}),\\ \dot{x}_{i,n_i}=u_i+f_{i,n_i}(x_i)+g_{i,n_i}(x_{i,n_i}(t-d_{i,n_i}))+\Delta_{i,n_i}(\bar{y}),\\ y_i=x_{i,1},\end{cases}\tag{1}$$

式中：$i=1,2,\cdots,N$；$l=1,2,\cdots,n_i-1$；$\bar{x}_{i,l}=[x_{i,1},x_{i,2},\cdots,x_{i,l}]^{\mathrm{T}}\in R^l$；$x_i=[x_{i,1},x_{i,2},\cdots,x_{i,n_i}]\in R^{n_i}$，表示第 i 个子系统的状态；$\bar{y}=[y_1,y_2,\cdots,y_N]^{\mathrm{T}}\in R^N$，表示第 i 个子系统的输出；$f_{i,l}(\cdot)$是未知的光滑非线性函数；$g_{i,l}(\cdot)$是未知的非线性时变函数；$d_{i,l}(t)$是未知的时间延迟，并且 $\dot{d}_{i,l}\leqslant\bar{d}_{i,l}<1$，$l=1,2,\cdots,n_i$；$\Delta_{i,l}(\cdot)$代表子系统之间的未知非线性互联项；$u_i$ 是在本文后续部分要设计的实际控制器。

此外，为实现控制目标，引入如下的假设和引理。

假设1：参考信号 $y_{i,r}(t)$及其导数 $y_{i,r}^{(i)}(t)$都是已知且有界的，其中 $i=1,2,\cdots,N$。

假设2：互联项 Δ_i 满足如下不等式

$$\|\Delta_i(\bar{y})\|\leqslant\sum_{j=1}^{N}\Gamma_{i,j}(|y_j|),\tag{2}$$

式中 $i=1,2,\cdots,N$。未知的非线性函数满足 $\breve{\Gamma}_{i,j}(y_j)=y_j\breve{\Gamma}_{i,j}^*(y_j)$、$\breve{\Gamma}_{i,j}(y_j)=y_j\breve{\Gamma}_{i,j}^*(y_j)$、$\bar{\Gamma}_{i,j}(y_j)=y_j\bar{\Gamma}_{i,j}^*(y_j)$，其中 $\breve{\Gamma}_{i,j}(y_j)$、$\breve{\Gamma}_{i,j}^*(y_j)$、$\bar{\Gamma}_{i,j}(y_j)$和 $\bar{\Gamma}_{i,j}^*(y_j)$都是已知的光滑函数。

引理1[24]：设 $\vartheta_1\in R^{n_{\vartheta,1}}$，$\vartheta_1\in R^{n_{\vartheta,2}}$，且 $g:R^{n_{\vartheta,1}\times n_{\vartheta,2}}\rightarrow R$ 是一个连续函数，可以得到 $|g(\vartheta_1,\vartheta_2)|\leqslant p_1(\vartheta_1)+p_2(\vartheta_2)$，其中 $p_1(\vartheta_1)\geqslant0$ 和 $p_2(\vartheta_2)\geqslant0$ 都是连续函数。

从假设2中可以得到时滞非线性函数 $g_{i,l}(\cdot)$满足如下不等式：

$$|g_{i,l}(x_{i,l}(t-d_{i,l}))|\leqslant\sum_{j=1}^{l}q_{i,l,j}(x_{i,l}(t-d_{i,l})),\tag{3}$$

式中 $q_{i,l,j}(x_{i,l}(t-d_{i,l}(t)))\geqslant0$，是未知的时滞函数。

注1：假设1是自适应反演控制方法的常见要求[25]；假设2是对于未知的互联项的一个常见条件[26]。

2　基于极限学习机的分散式自适应跟踪控制设计

针对互联非线性时滞系统(1)，基于动态面控制技术，给出如下坐标转换方程：

$$z_{i,1}=y_i-y_{i,r},\quad z_{i,l}=x_{i,l}-\omega_{i,l},\quad \chi_{i,l}=\omega_{i,l}-\alpha_{i,l-1},\tag{4}$$

式中：$l=2,3,\cdots,n_i$；$\alpha_{i,l}$ 是虚拟控制律，这将在本文后续进行设计；$\omega_{i,l}$ 为相应的滤波器信号，其作用主要是用来避免 $\alpha_{i,l-1}$ 的微分运算。因此，基于动态面控制技术的递归设计过程如下。

第1步　根据式(1)和式(4)，可以得到 $z_{i,1}$ 的导数如下：

$$\begin{aligned}\dot{z}_{i,1}=\dot{y}_i-\dot{y}_{i,r}=x_{i,2}+f_{i,1}+g_{i,1}(x_{i,1}(t-d_{i,1}))+\Delta_{i,1}-\dot{y}_{i,r}=\\ z_{i,2}+\chi_{i,2}+\alpha_{i,1}+f_{i,1}+g_{i,1}(x_{i,1}(t-d_{i,1}))+\Delta_{i,1}-\dot{y}_{i,r}。\end{aligned}$$

考虑如下李雅普诺夫函数：

$$V_{i,1}=\frac{1}{2}z_{i,1}^2+\frac{1}{2}\chi_{i,2}^2+\frac{1}{2\sigma_{i,1}}\tilde{\theta}_{i,1}^2+H_{i,1},\tag{5}$$

式中：$\sigma_{i,1}>0$ 是常数；$\theta_{i,1}=\|W_{i,1}\|^2$；$\tilde{\theta}_{i,1}=\theta_{i,1}-\hat{\theta}_{i,1}$，其中 $\hat{\theta}_{i,1}$ 是 $\theta_{i,1}$ 的估计值；Lyapunov-Krasovskii 函数 $H_{i,1}$ 为

$$H_{i,1}=\frac{1}{1-\bar{d}_{i,1}}\mathrm{e}^{-\tau(t-d_{i,1})}\int_{t-d_{i,1}}^{t}\mathrm{e}^{\tau s}q_{i,1,1}^2(x_{i,1}(s))\mathrm{d}s,$$

其中 τ 是正设计参数。

对式(5)求导，可得：

$$\begin{aligned}\dot{V}_{i,1}=\dot{z}_{i,1}z_{i,1}+\dot{\chi}_{i,2}\chi_{i,2}-\frac{1}{\sigma_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}+\dot{H}_{i,1}=\\ z_{i,1}(z_{i,2}+\chi_{i,2}+\alpha_{i,1}+f_{i,1}+g_{i,1}(x_{i,1}(t-d_{i,1}))+\Delta_{i,1}-\dot{y}_{i,r})+\\ \dot{\chi}_{i,2}\chi_{i,2}-\frac{1}{\sigma_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}+\dot{H}_{i,1},\end{aligned}\tag{6}$$

式中:

$$\dot{H}_{i,1}=-\frac{\tau(1-\dot{d}_{i,1}(t))}{1-\bar{d}_{i,1}}\mathrm{e}^{-\tau(t-d_{i,1}(t))}\int_{t-d_{i,1}(t)}^{t}\mathrm{e}^{\tau s}q_{i,1,1}^{2}(x_{i,1}(s))\mathrm{d}s+$$
$$\frac{\mathrm{e}^{-\tau(t-d_{i,1}(t))}}{1-\bar{d}_{i,1}}\left[\mathrm{e}^{\tau t}q_{i,1,1}^{2}(t)-(1-\dot{d}_{i,1}(t))\mathrm{e}^{\tau(t-d_{i,1}(t))}q_{i,1,1}^{2}(x_{i,1}(t-d_{i,1}(t)))\right],\tag{7}$$

使用 Young's 不等式,可得:

$$z_{i,1}\chi_{i,2}\leqslant\frac{1}{2}z_{i,1}^{2}+\frac{1}{2}\chi_{i,2}^{2},\tag{8}$$

$$z_{i,1}g_{i,1}(x_{i,1}(t-d_{i,1}))\leqslant|z_{i,1}|q_{i,1,1}^{2}(x_{i,1}(t-d_{i,1}))\leqslant\frac{1}{4}z_{i,1}^{2}+q_{i,1,1}^{2}(x_{i,1}(t-d_{i,1})),\tag{9}$$

$$z_{i,1}\Delta_{i,1}\leqslant\frac{1}{4}z_{i,1}^{2}+\|\Delta_{i}(\bar{y})\|^{2}。\tag{10}$$

由于 $\dot{d}_{i,1}(t)\leqslant\bar{d}_{i,1}<1,d_{i,1}(t)\leqslant d_{i}$,可得:

$$-(1-\dot{d}_{i,1}(t))\leqslant-(1-\bar{d}_{i,1}),$$

以及

$$\mathrm{e}^{\tau d_{i,1}(t)}\leqslant\mathrm{e}^{\tau d_{i}},$$

将式(7)—式(10)代入式(6),可得:

$$\dot{V}_{i,1}\leqslant z_{i,1}(z_{i,2}+\alpha_{i,1}+f_{i,1}+z_{i,1}-\dot{y}_{i,r})+\frac{1}{2}\chi_{i,2}^{2}+\chi_{i,2}\dot{\chi}_{i,2}-\frac{1}{\sigma_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}-$$
$$\tau(1-\bar{d}_{i,1})H_{i,1}+\frac{\mathrm{e}^{\tau d_{i}}}{1-\bar{d}_{i,1}}q_{i,1,1}^{2}(x_{i,1}(t))+\|\Delta_{i}(\bar{y})\|^{2}。\tag{11}$$

定义如下函数:

$$\Omega_{i,1}=\frac{\mathrm{e}^{\tau d_{i}}}{1-\bar{d}_{i,1}}q_{i,1,1}^{2}(x_{i,1}(t))。$$

通过在式(11)中同时加减 $k_{i,1}z_{i,1}^{2}\Omega_{i,1}$,可得:

$$\dot{V}_{i,1}\leqslant z_{i,1}(z_{i,2}+\alpha_{i,1}+\boldsymbol{\Psi}_{i,1}+z_{i,1}-\dot{y}_{i,r})+\frac{1}{2}\chi_{i,2}^{2}+\chi_{i,2}\dot{\chi}_{i,2}-\frac{1}{\sigma_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}-$$
$$\tau(1-\bar{d}_{i,1})H_{i,1}+(1-k_{i,1}z_{i,1}^{2})\Omega_{i,1}+\|\Delta_{i}(\bar{y})\|^{2},\tag{12}$$

式中:$k_{i,1}>0$,是常数;$\boldsymbol{\Psi}_{i,1}=f_{i,1}+k_{i,1}z_{i,1}\Omega_{i,1}$,是一个未知函数。

针对式(12)中的未知非线性函数 $\boldsymbol{\Psi}_{i,1}$,利用极限学习机的近似特性,即一定存在最优输出权重参数 W^{*} 使得 $\boldsymbol{\Psi}_{i,1}$ 满足:

$$\boldsymbol{\Psi}_{i,1}=S_{i,1}(x_{i,1},\bar{\omega},d)W_{i,1}^{*}+\delta_{i,1}(x),\tag{13}$$

其中 $\delta_{i,1}(x)$ 是近似误差,满足 $|\delta_{i,1}(x)|\leqslant\varepsilon_{i,1},\varepsilon_{i,1}>0$。定义 $\theta_{i}=\|W_{i,1}^{*2}\|,i=1,2,\cdots,N$。

将式(13)代入式(12)可得:

$$\dot{V}_{i,1}\leqslant z_{i,1}(z_{i,2}+\alpha_{i,1}+S_{i,1}(x,\bar{\omega},d)W_{i,1}^{*}+\delta_{i,1}(x)+z_{i,1}-\dot{y}_{i,r})+\frac{1}{2}\chi_{i,2}^{2}+\chi_{i,2}\dot{\chi}_{i,2}-$$
$$\frac{1}{\sigma_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}-\tau(1-\bar{d}_{i,1})H_{i,1}+(1-k_{i,1}z_{i,1}^{2})\Omega_{i,1}+\|\Delta_{i}(\bar{y})\|^{2}。\tag{14}$$

使用 Young's 不等式,可得:

$$z_{i,1}S_{i,1}(x,\bar{\omega},d)W_{i,1}^{*}\leqslant\frac{\theta}{2a_{i,1}}z_{i,1}^{2}\boldsymbol{S}_{i,1}^{\mathrm{T}}\boldsymbol{S}_{i,1}+\frac{1}{2}a_{i,1},\tag{15}$$

$$z_{i,1}\delta_{i,1}(x)\leqslant\frac{1}{2}z_{i,1}^{2}+\frac{1}{2}\varepsilon_{i,1}^{2}。\tag{16}$$

将式(15)、式(16)代入式(14)可得:

$$\dot{V}_{i,1}\leqslant z_{i,1}(z_{i,2}+\alpha_{i,1}+\frac{3}{2}z_{i,1}+\frac{\theta_{i,1}}{2a_{i,1}}z_{i,1}\boldsymbol{S}_{i,1}^{\mathrm{T}}\boldsymbol{S}_{i,1}-\dot{y}_{i,r})+\frac{1}{2}\chi_{i,2}^{2}+\chi_{i,2}\dot{\chi}_{i,2}-$$

$$\frac{1}{\sigma_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}-\tau(1-\bar{d}_{i,1})H_{i,1}+(1-k_{i,1}z_{i,1}^{2})\Omega_{i,1}+\|\Delta_i(\bar{y})\|^2+\frac{1}{2}a_{i,1}+\frac{1}{2}\varepsilon_{i,1}^{2}。\tag{17}$$

根据式(17),设计虚拟控制律:

$$\alpha_{i,1}=-c_{i,1}z_{i,1}-\frac{3}{2}z_{i,1}-\frac{\hat{\theta}_{i,1}}{2a_{i,1}}z_{i,1}\boldsymbol{S}_{i,1}^{\mathrm{T}}\boldsymbol{S}_{i,1}+\dot{y}_{i,r}-z_{i,1}\varphi_i(\bar{y}),\tag{18}$$

式中:$c_{i,1}$ 是一个正设计常数;$\varphi_i(\bar{y})$是一个设计函数,将在本文后续进行定义。

自适应参数更新算法选择为

$$\dot{\hat{\theta}}_{i,1}=\frac{1}{2a_{i,1}}\sigma_{i,1}z_{i,1}^{2}\boldsymbol{S}_{i,1}^{\mathrm{T}}\boldsymbol{S}_{i,1}-c_{\theta}\hat{\theta}_{i,1}。\tag{19}$$

一阶低通滤波器算法为

$$l_{i,2}\dot{\omega}_{i,1}+\omega_{i,1}=\alpha_{i,1},\omega_{i,2}(0)=\alpha_{i,1}(0),\tag{20}$$

式中:$l_{i,2}$ 是正常数;$\omega_{i,2}$ 是低通滤波器的输出;$\alpha_{i,2}$ 是相应的输入。

通过 Young's 不等式,可得:

$$\chi_{i,2}\dot{\chi}_{i,2}\leqslant-\frac{1}{l_{i,1}}\chi_{i,1}^{2}+\frac{1}{2}\chi_{i,2}^{2}+\frac{1}{2}L_{i,1}^{2},\tag{21}$$

其中$|\dot{\alpha}_{i,2}|\leqslant L_{i,1}^{2}$,是有界的。

将式(18)、式(19)和式(21)代入式(17),可得:

$$\dot{V}_{i,1}\leqslant-c_{i,1}z_{i,2}^{2}+z_{i,1}z_{i,2}-(\frac{1}{l_{i,2}}-1)\chi_{i,2}^{2}+\frac{c_{\theta}}{\sigma_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}-\tau(1-\bar{d}_{i,1})H_{i,1}+$$
$$(1-k_{i,1}z_{i,1}^{2})\Omega_{i,1}+\|\Delta_i(\bar{y})\|^2+\frac{1}{2}a_{i,1}+\frac{1}{2}\varepsilon_{i,1}^{2}+\frac{1}{2}L_{i,1}^{2}-z_{i,1}^{2}\varphi_i(\bar{y})。$$

第 l 步($2\leqslant l\leqslant n_i-1$) 与第 1 步相似,根据式(1)和式(4),可以得到 $z_{i,l}$ 的导数如下:

$$\dot{z}_{i,l}=\dot{x}_{i,l}-\dot{\omega}_{i,l}=$$
$$x_{i,l+1}+f_{i,1}+g_{i,1}(x_i(t-d_{i,l}))+\Delta_{i,l}-\dot{\omega}_{i,l}=$$
$$z_{i,l+1}+\chi_{i,l+1}+\alpha_{i,l}+f_{i,1}+g_{i,1}(x_i(t-d_{i,l}))+\Delta_{i,l}-\dot{\omega}_{i,l}。$$

选择如下李雅普诺夫函数:

$$V_{i,1}=V_{i,l-1}+\frac{1}{2}z_{i,l}^{2}+\frac{1}{2}\chi_{i,l+1}^{2}+\frac{1}{2\sigma_{i,1}}\tilde{\theta}_{i,l}^{2}+H_{i,l},$$

式中:$\sigma_{i,l}>0$,是常数;$\theta_{i,l}=\|W_{i,l}^{*}\|^2$,$\tilde{\theta}_{i,l}=\theta_{i,l}-\hat{\theta}_{i,l}$,$\hat{\theta}_{i,l}$ 是 $\theta_{i,l}$ 的估计值;Lyapunov-Krasovskii 函数 $H_{i,l}$ 形式为

$$H_{i,l}=\sum_{j=1}^{l}\frac{1}{1-\bar{d}_{i,l}}\mathrm{e}^{-\tau(t-d_{i,j})}\int_{t-d_{i,l}}^{t}\mathrm{e}^{\tau s}q_{i,l,j}^{2}(x_{i,l}(s))\mathrm{d}s,$$

其中 τ 是正设计常数。

对 $\dot{V}_{i,l}$ 求导,可得:

$$\dot{V}_{i,l}=\dot{V}_{i,l-1}+z_{i,l}\dot{z}_{i,l}+\chi_{i,l+1}\dot{\chi}_{i,l+1}-\frac{1}{\sigma_{i,l}}\tilde{\theta}_{i,l}\dot{\hat{\theta}}_{i,l}+\dot{H}_{i,l}=$$
$$\dot{V}_{i,l-1}+z_{i,l}(z_{i,l+1}+\chi_{i,l+1}+\alpha_{i,l}+f_{i,l}+g_{i,l}(x_{i,l}(t-d_{i,l}))+$$
$$\Delta_{i,l}-\dot{\omega}_{i,l})+\chi_{i,l+1}\dot{\chi}_{i,l+1}-\frac{1}{\sigma_{i,l}}\tilde{\theta}_{i,l}\dot{\hat{\theta}}_{i,l}+\dot{H}_{i,l},\tag{22}$$

式中

$$\dot{H}_{i,l}=-\tau(t-\dot{d}_{i,l}(t))H_{i,l}+\sum_{j=1}^{l}\frac{1-\dot{d}_{i,l}(t)}{1-d_{i,l}}q_{i,l,j}^{2}(x_{i,l}(t-d_{i,l}(t)))。\tag{23}$$

使用 Young's 不等式,可得:

$$z_{i,l}\chi_{i,l+1}\leqslant\frac{1}{2}z_{i,l}^{2}+\frac{1}{2}\chi_{i,l+1}^{2},\tag{24}$$

$$z_{i,l}g_{i,l}(x_{i,l}(t-d_{i,l}))\leqslant|z_{i,l}|\sum_{j=1}^{l}q_{i,l,j}(x_{i,l}(t-d_{i,l}))\leqslant\frac{1}{4}z_{i,l}^2+\sum_{j=1}^{l}q_{i,l,j}^2(x_{i,l}(t-d_{i,l})),\tag{25}$$

$$z_{i,l}\Delta_{i,l}\leqslant\frac{1}{4}z_{i,l}^2+\|\Delta_i(\bar{y})\|^2,\tag{26}$$

$$\chi_{i,l+1}\dot{\chi}_{i,l+1}\leqslant\frac{1}{l_{i,l+1}}\chi_{i,l+1}^2+\frac{1}{2}\chi_{i,l+1}^2+\frac{1}{2}L_{i,l}^2。\tag{27}$$

将式(23)—式(27)代入式(22)可得：

$$\dot{V}_{i,l}\leqslant\dot{V}_{i,l-1}+z_{i,l}(z_{i,l+1}+\frac{l}{4}z_{i,l}+\frac{3}{4}z_{i,l}+\alpha_{i,l}+f_{i,l}-\dot{\omega}_{i,l})-(\frac{1}{l_{i,l+1}}-1)\chi_{i,l+1}^2-\frac{1}{\sigma_{i,l}}\tilde{\theta}_{i,l}\dot{\hat{\theta}}_{i,l}-\tau(1-\dot{d}_{i,l}(t))H_{i,l}+\sum_{j=1}^{l}\frac{e^{\tau d_{i,l}(t)}}{1-\bar{d}_{i,l}}q_{i,l,j}^2(x_{i,l}(t))+\|\Delta_i(\bar{y})\|^2+\frac{1}{2}L_{i,l}^2。\tag{28}$$

定义如下函数：

$$\Omega_{i,l}=\sum_{j=1}^{l}\frac{e^{\tau d_{i,l(t)}}}{1-\bar{d}_{i,l}}q_{i,l,j}^2(x_{i,l}(t))。\tag{29}$$

通过在式(28)中同时加减 $k_{i,l}z_{i,l}^2\Omega_{i,l}$，可得：

$$\dot{V}_{i,l}\leqslant\dot{V}_{i,l-1}+z_{i,l}(z_{i,l+1}+\frac{l}{4}z_{i,l}+\frac{3}{4}z_{i,l}+\alpha_{i,l}+\Psi_{i,l}-\dot{\omega}_{i,l})-(\frac{1}{l_{i,l+1}}-1)\chi_{i,l+1}^2-\frac{1}{\sigma_{i,l}}\tilde{\theta}_{i,l}\dot{\hat{\theta}}_{i,l}-\tau(1-\dot{d}_{i,l}(t))H_{i,l}+(1-k_{i,l}z_{i,l}^2)\Omega_{i,l}+\|\Delta_i(\bar{y})\|^2+\frac{1}{2}L_{i,l}^2,\tag{30}$$

式中：$k_{i,l}$ 是常数；$\Psi_{i,l}=f_{i,l}+k_{i,l}z_{i,l}\Omega_{i,l}$，是一个未知函数。

利用极限学习机的近似特性估计式(30)中的未知非线性函数 $\Psi_{i,l}$，即

$$\Psi_{i,l}=S_{i,l}(x,\bar{\omega},d)W_{i,l}^*+\delta_{i,l}(x),\tag{31}$$

式中 $\delta_{i,l}(x)$是近似误差，且$|\delta_{i,l}(x)|\leqslant\varepsilon_{i,l}$，$\varepsilon_{i,l}>0$。

将式(31)代入式(30)可得：

$$\dot{V}_{i,l}\leqslant\dot{V}_{i,l-1}+z_{i,l}(z_{i,l+1}+\frac{l}{4}z_{i,l}+\frac{3}{4}z_{i,l}+\alpha_{i,l}+S_{i,l}(x,\bar{\omega},d)W_{i,l}^*+\delta_{i,l}(x)-\dot{\omega}_{i,l})-\frac{1}{\sigma_{i,l}}\tilde{\theta}_{i,l}\dot{\hat{\theta}}_{i,l}-(\frac{1}{l_{i,l+1}}-1)\chi_{i,l+1}^2-\tau(1-\dot{d}_{i,l}(t))H_{i,l}+(1-k_{i,l}z_{i,l}^2)\Omega_{i,l}+\|\Delta_i(\bar{y})\|^2+\frac{1}{2}L_{i,l}^2。\tag{32}$$

使用 Young's 不等式，可得：

$$z_{i,l}S_{i,l}(x,\bar{\omega},d)W_{i,l}^*\leqslant\frac{\theta_{i,l}}{2a_{i,l}}z_{i,l}^2\boldsymbol{S}_{i,l}^{\mathrm{T}}\boldsymbol{S}_{i,l}+\frac{1}{2}a_{i,l},\tag{33}$$

$$z_{i,l}\delta_{i,l}(x)\leqslant\frac{1}{2}z_{i,l}^2+\frac{1}{2}\varepsilon_{i,l}^2,\tag{34}$$

将式(33)、式(34)代入式(32)可得：

$$\dot{V}_{i,l}\leqslant\dot{V}_{i,l-1}+z_{i,l}(z_{i,l+1}+\frac{l}{4}z_{i,l}+\frac{5}{4}z_{i,l}+\alpha_{i,l}-\dot{\omega}_{i,l})-(\frac{1}{l_{i,l+1}}-1)\chi_{i,l+1}^2-\frac{1}{\sigma_{i,l}}\tilde{\theta}_{i,l}\dot{\hat{\theta}}_{i,l}-\tau(1-\dot{d}_{i,l}(t))H_{i,l}+(1-k_{i,l}z_{i,l}^2)\Omega_{i,l}+\frac{\theta_{i,l}}{2a_{i,l}}z_{i,l}^2\boldsymbol{S}_{i,l}^{\mathrm{T}}\boldsymbol{S}_{i,l}+\|\Delta_i(\bar{y})\|^2+\frac{1}{2}a_{i,l}+\frac{1}{2}\varepsilon_{i,l}^2+\frac{1}{2}L_{i,l}^2。\tag{35}$$

根据式(35)，设计虚拟控制律 $\alpha_{i,l}$ 如下：

$$\alpha_{i,l}=-c_{i,l}z_{i,l}-z_{i,l-1}-\frac{5}{4}z_{i,l}-\frac{l}{4}z_{i,l}-\frac{\hat{\theta}_{i,l}}{2a_{i,l}}z_{i,l}\boldsymbol{S}_{i,l}^{\mathrm{T}}\boldsymbol{S}_{i,l}+\dot{\omega}_{i,l}。\tag{36}$$

自适应参数更新算法选择为

$$\dot{\hat{\theta}}_{i,l}=\frac{1}{2a_{i,l}}\sigma_{i,l}z_{i,l}^2\boldsymbol{S}_{i,l}^{\mathrm{T}}\boldsymbol{S}_{i,l}-c_{\theta}\hat{\theta}_{i,l}。\tag{37}$$

在第$(l-1)$步中，可得：

$$\dot{V}_{i,l-1}\leqslant-\sum_{j=1}^{l-1}c_{i,j}z_{i,j}^{2}+z_{i,l-1}z_{i,l}-\sum_{j=2}^{l}(\frac{1}{\iota_{i,j}}-1)\chi_{i,j}^{2}+\sum_{j=1}^{l-1}\frac{c_{\theta}}{\sigma_{i,j}}\tilde{\theta}_{i,j}\hat{\theta}_{i,j}-\sum_{j=1}^{l-1}\tau(1-\bar{d}_{i,j})H_{i,j}+\sum_{j=1}^{l-1}(1-k_{i,j}z_{i,j}^{2})\Omega_{i,j}+\frac{1}{2}\sum_{j=1}^{l-1}(a_{i,j}+\varepsilon_{i,j}^{2}+L_{i,j}^{2})+l\|\Delta_{i}(\bar{y})\|^{2}-z_{i,1}^{2}\varphi_{i}(\bar{y})。\tag{38}$$

将式(36)—式(38)代入式(35)，可得：

$$\dot{V}_{i,l}\leqslant-\sum_{j=1}^{l}c_{i,j}z_{i,j}^{2}+z_{i,l}z_{i,l+1}-\sum_{j=2}^{l+1}(\frac{1}{l_{i,j}}-1)\chi_{i,j}^{2}+\sum_{j=1}^{l}\frac{c_{\theta}}{\sigma_{i,j}}\tilde{\theta}_{i,j}\hat{\theta}_{i,j}-\sum_{j=1}^{l}\tau(1-\bar{d}_{i,j})H_{i,j}+\sum_{j=1}^{l}(1-k_{i,j}z_{i,j}^{2})\Omega_{i,j}+\frac{1}{2}\sum_{j=1}^{l}(a_{i,j}+\varepsilon_{i,j}^{2}+L_{i,j}^{2})+l\|\Delta_{i}(\bar{y})\|^{2}-z_{i,1}^{2}\varphi_{i}(\bar{y})。$$

第 n_i 步　根据式(1)和式(4)，可以得到 z_{i,n_i} 的导数如下：

$$\dot{z}_{i,n_i}=\dot{x}_{i,n_i}-\dot{\omega}_{i,n_i}=u_i+f_{i,n_i}+g_{i,n_i}(x_{i,n_i}(t-d_{i,n_i}))+\Delta_{i,n_i}-\dot{\omega}_{i,n_i}。$$

考虑如下李雅普诺夫函数

$$V_{i,n_i}=V_{i,n_i-1}+\frac{1}{2}z_{i,n_i}^{2}+\frac{1}{2\sigma_{i,n_i}}\tilde{\theta}_{i,n_i}^{2}+H_{i,n_i},$$

式中：$\sigma_{i,n_i}>0$，是常数；$\theta_{i,n_i}=\|W_{i,n_i}^{*}\|^{2}$，$\tilde{\theta}_{i,n_i}=\theta_{i,n_i}-\hat{\theta}_{i,n_i}$，$\hat{\theta}_{i,n_i}$ 是 θ_{i,n_i} 的估计值；Lyapunov-Krasovskii 函数 H_{i,n_i} 形式如下：

$$H_{i,n_i}=\sum_{j=1}^{n_i}\frac{1}{1-\bar{d}_{i,n_i}}\mathrm{e}^{-\tau(t-d_{i,n_i})}\int_{t-d_{i,n_i}}^{t}\mathrm{e}^{\tau s}q_{i,n_i,j}^{2}(x_{i,n_i}(s))\mathrm{d}s,$$

其中 τ 是正设计参数。

对 V_{i,n_i} 求导，可得：

$$\dot{V}_{i,n_i}=\dot{V}_{i,n_i-1}+z_{i,n_i}\dot{z}_{i,n_i}-\frac{1}{\sigma_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i}+\dot{H}_{i,n_i}=\dot{V}_{i,n_i-1}+z_{i,n_i}(u_i+f_{i,n_i}+g_{i,n_i}(x_{i,n_i}(t-d_{i,n_i}))+\Delta_{i,n_i}-\dot{\omega}_{i,n_i})-\frac{1}{\sigma_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i}+\dot{H}_{i,n_i},\tag{39}$$

式中 $\dot{H}_{i,n_i}$ 为

$$\dot{H}_{i,n_i}=-\tau(1-\dot{d}_{i,n_i})H_{i,n_i}+\sum_{j=1}^{n_i}\frac{\mathrm{e}^{\tau d_{i,n_i}(t)}}{1-\bar{d}_{i,n_i}}q_{i,n_i,j}^{2}(x(t))-\sum_{j=1}^{n_i}\frac{1-\dot{d}_{i,n_i}(t)}{1-\bar{d}_{i,n_i}}q_{i,n_i,j}^{2}(x_{i,n_i}(t-d_{i,n_i}))。\tag{40}$$

使用 Young's 不等式，可得：

$$z_{i,n_i}g_{i,n_i}(x_{i,n_i}(t-d_{i,n_i}))\leqslant|z_{i,n_i}|\sum_{j=1}^{n_i}q_{i,n_i,j}(x_{i,n_i}(t-d_{i,n_i}))\leqslant\frac{n_i}{4}z_{i,n_i}^{2}+\sum_{j=1}^{n_i}q_{i,n_i,j}^{2}(x_{i,n_i}(t-d_{i,n_i})),\tag{41}$$

$$z_{i,n_i}\Delta_{i,n_i}\leqslant\frac{1}{4}z_{i,n_i}^{2}+\|\Delta_i(\bar{y})\|^{2}。\tag{42}$$

将式(40)—式(42)代入式(39)可得：

$$\dot{V}_{i,n_i}\leqslant\dot{V}_{i,n_i-1}+z_{i,n_i}(u_i+f_{i,n_i}+\frac{n_i}{4}z_{i,n_i}+\frac{1}{4}z_{i,n_i}-\dot{\omega}_{i,n_i})-\frac{1}{\sigma_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i}-\tau(1-\dot{d}_{i,n_i}(t))H_{i,n_i}+\|\Delta_i(\bar{y})\|^{2}+\sum_{j=1}^{n_i}\frac{\mathrm{e}^{\tau d_{i,n_i}(t)}}{1-\bar{d}_{i,n_i}}q_{i,n_i,j}^{2}(x_{i,n_i}(t))。\tag{43}$$

定义如下函数

$$\Omega_{i,n_i}=\sum_{j=1}^{n_i}\frac{\mathrm{e}^{\tau d_{n_i}}}{1-\bar{d}_{i,n_i}}q_{i,n_i,j}^{2}(x_{i,n_i}(t)),$$

通过在式(43)中同时加减 $k_{i,n_i}z_{i,n_i}^{2}\Omega_{i,n_i}$，可得：

$$\dot{V}_{i,n_i} \leqslant \dot{V}_{i,n_i-1} + z_{i,n_i}(u_i + \frac{n_i}{4}z_{i,n_i} + \frac{1}{4}z_{i,n_i} + \Psi_{i,n_i} - \dot{\omega}_{i,n_i}) - \frac{1}{\sigma_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i} - \tau(1-\dot{d}_{i,n_i}(t))H_{i,n_i} + (1-k_{i,n_i}z_{i,n_i}^2)\Omega_{i,n_i} + \|\Delta_i(\bar{y})\|^2, \tag{44}$$

式中：k_{i,n_i} 是常数；$\Psi_{i,n_i}=f_{i,n_i}+k_{i,n_i}z_{i,n_i}\Omega_{i,n_i}$，是一个未知函数。

利用极限学习机的近似特性估计式(44)中的未知非线性函数 Ψ_{i,n_i}，即

$$\Psi_{i,n_i} = S_{i,n_i}(x,\bar{\omega},d)W_{i,n_i}^* + \delta_{i,n_i}(x), \tag{45}$$

式中 $\delta_{i,n_i}(x)$是近似误差，且$|\delta_{i,n_i}(x)|\leqslant\varepsilon_{i,n_i}$，$\varepsilon_{i,n_i}>0$。

将式(45)代入式(44)，可得：

$$\dot{V}_{i,n_i} \leqslant \dot{V}_{i,n_i-1} + z_{i,n_i}(u_i + \frac{n_i}{4}z_{i,n_i} + \frac{1}{4}z_{i,n_i} + S_{i,n_i}(x,\bar{\omega},d)W_{i,n_i}^* + \delta_{i,n_i}(x) - \dot{\omega}_{i,n_i}) - \frac{1}{\sigma_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i} - \tau(1-\dot{d}_{i,n_i}(t))H_{i,n_i} + (1-k_{i,n_i}z_{i,n_i}^2)\Omega_{i,n_i} + \|\Delta_i(\bar{y})\|^2。 \tag{46}$$

使用 Young's 不等式，可得：

$$z_{i,n_i}S_{i,n_i}(x,\bar{\omega},d)W_{i,n_i}^* \leqslant \frac{\theta_{i,n_i}}{2a_{i,n_i}}z_{i,n_i}^2\boldsymbol{S}_{i,n_i}^{\mathrm{T}}\boldsymbol{S}_{i,n_i} + \frac{1}{2}a_{i,n_i}, \tag{47}$$

$$z_{i,n_i}\delta_{i,n_i}(x) \leqslant \frac{1}{2}z_{i,n_i}^2 + \frac{1}{2}\varepsilon_{i,n_i}^2。 \tag{48}$$

将式(47)、式(48)代入式(46)可得：

$$\dot{V}_{i,n_i} \leqslant \dot{V}_{i,n_i-1} + z_{i,n_i}(u_i + \frac{n_i}{4}z_{i,n_i} + \frac{3}{4}z_{i,n_i} - \dot{\omega}_{i,n_i}) - \frac{1}{\sigma_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i} - \tau(1-\dot{d}_{i,n_i}(t))H_{i,n_i} + (1-k_{i,n_i}z_{i,n_i}^2)\Omega_{i,n_i} + \frac{\theta_{i,n_i}}{2a_{i,n_i}}z_{i,n_i}^2\boldsymbol{S}_{i,n_i}^{\mathrm{T}}\boldsymbol{S}_{i,n_i} + \|\Delta_i(\bar{y})\|^2 + \frac{1}{2}a_{i,n_i} + \frac{1}{2}\varepsilon_{i,n_i}^2。 \tag{49}$$

根据式(49)，设计实际控制器为

$$u_i = -c_{i,n_i}z_{i,n_i} - z_{i,n_i-1} - \frac{n_i}{4}z_{i,n_i} - \frac{3}{4}z_{i,n_i} - \frac{\hat{\theta}_{i,n_i}}{2a_{i,n_i}}z_{i,n_i}\boldsymbol{S}_{i,n_i}^{\mathrm{T}}\boldsymbol{S}_{i,n_i} + \dot{\omega}_{i,n_i}, \tag{50}$$

式中 c_{i,n_i} 是一个正设计常数。

自适应参数更新算法选择为

$$\dot{\hat{\theta}}_{i,n_i} = \frac{1}{2a_{i,n_i}}\sigma_{i,n_i}z_{i,n_i}^2\boldsymbol{S}_{i,n_i}^{\mathrm{T}}\boldsymbol{S}_{i,n_i} - c_\theta\hat{\theta}_{i,n_i}。 \tag{51}$$

在第(n_i-1)步中，可得：

$$\dot{V}_{i,n_i-1} \leqslant -\sum_{j=1}^{n_i-1}c_{i,j}z_{i,j}^2 + z_{i,n_i-1}z_{i,n_i} - \sum_{j=1}^{n_i}(\frac{1}{l_{i,j}}-1)\chi_{i,j}^2 + \sum_{j=1}^{n_i-1}\frac{c_\theta}{\sigma_{i,j}}\tilde{\theta}_{i,j}\hat{\theta}_{i,j} - \sum_{j=1}^{n_i-1}\tau(1-\bar{d}_{i,j})H_{i,j} + \sum_{j=1}^{n_i-1}(1-k_{i,j}z_{i,j}^2)\Omega_{i,j} + (n_i-1)\|\Delta_i(\bar{y})\|^2 + \frac{1}{2}\sum_{j=1}^{n_i-1}(a_{i,j}+\varepsilon_{i,j}^2+L_{i,j}^2)。 \tag{52}$$

将式(50)—式(52)代入式(49)可得：

$$\dot{V}_{i,n_i} \leqslant -\sum_{j=1}^{n_i}c_{i,j}z_{i,j}^2 - \sum_{j=1}^{n_i}(\frac{1}{l_{i,j}}-1)\chi_{i,j}^2 + \sum_{j=1}^{n_i}\frac{c_\theta}{\sigma_{i,j}}\tilde{\theta}_{i,j}\hat{\theta}_{i,j} - \sum_{j=1}^{n_i}\tau(1-\bar{d}_{i,j})H_{i,j} + \sum_{j=1}^{n_i}(1-k_{i,j}z_{i,j}^2)\Omega_{i,j} + \frac{1}{2}\sum_{j=1}^{n_i}(a_{i,j}+\varepsilon_{i,j}^2) + \frac{1}{2}\sum_{j=1}^{n_i-1}L_{i,j}^2 + n_i\|\Delta_i(\bar{y})\|^2 - z_{i,1}^2\varphi_i(\bar{y})。 \tag{53}$$

根据假设 2，互联项 $\Delta_i(\bar{y})$可以重写成如下形式：

$$\|\Delta_i(\bar{y})\|^2 \leqslant (\sum_{j=1}^{N}\Gamma_{i,j}(|y_j|))^2 \leqslant N\sum_{j=1}^{N}(2\Gamma_{i,j}^2(2|z_{i,1}|) + 2\Gamma_{i,j}^2(2|y_{rj}|)) \leqslant N\sum_{j=1}^{N}8z_{i,1}^2(\breve{\Gamma}_{j,i}^*(2|z_{i,1}|) + \bar{\Gamma}_{j,i}^*(2|z_{i,1}|))^2 + B_i, \tag{54}$$

式中 $B_i \geqslant 2N\sum_{j=1}^{N}\Gamma_{j,i}^{2}(2\mid y_{rj}\mid)$，是正常数。

为了处理互联项，定义如下函数：

$$\varphi_i(\bar{y})=8n_i\sum_{j=1}^{N}N(\breve{\Gamma}_{j,i}^{*}(2\mid z_{i,1}\mid)+\bar{\Gamma}_{j,i}^{*}(2\mid z_{i,1}\mid))^2。\tag{55}$$

将式(54)和式(55)代入式(53)，可得：

$$\dot{V}_{i,n_i}\leqslant-\sum_{j=1}^{n_i}c_{i,j}z_{i,j}^2-\sum_{j=1}^{n_i}\left(\frac{1}{l_{i,j}}-1\right)\chi_{i,j}^2+\sum_{j=1}^{n_i}\frac{c_\theta}{\sigma_{i,j}}\tilde{\theta}_{i,j}\hat{\theta}_{i,j}-\sum_{j=1}^{n_i}\tau(1-\bar{d}_{i,j})H_{i,j}+\sum_{j=1}^{n_i}(1-k_{i,j}z_{i,j}^2)\Omega_{i,j}+\frac{1}{2}\sum_{j=1}^{n_i}(a_{i,j}+\varepsilon_{i,j}^2)+\frac{1}{2}\sum_{j=1}^{n_i-1}L_{i,j}^2+B_in_i。\tag{56}$$

使用Young′s不等式，可得：

$$\sum_{j=1}^{n_i}\frac{c_\theta}{\sigma_{i,j}}\tilde{\theta}_{i,j}\hat{\theta}_{i,j}\leqslant-\sum_{j=1}^{n_i}\frac{c_\theta}{2\sigma_{i,j}}\tilde{\theta}_{i,j}^2+\sum_{j=1}^{n_i}\frac{c_\theta}{2\sigma_{i,j}}\theta_{i,j}^2。\tag{57}$$

将式(57)代入式(56)，可得：

$$\dot{V}_{i,n_i}\leqslant-\sum_{j=1}^{n_i}c_{i,j}z_{i,j}^2-\sum_{j=1}^{n_i}(\frac{1}{l_{i,j}}-1)\chi_{i,j}^2-\sum_{j=1}^{n_i}\frac{c_\theta}{2\sigma_{i,j}}\tilde{\theta}_{i,j}^2-\sum_{j=1}^{n_i}\tau(1-\bar{d}_{i,j})H_{i,j}+\sum_{j=1}^{n_i}(1-k_{i,j}z_{i,j}^2)\Omega_{i,j}+\frac{1}{2}\sum_{j=1}^{n_i}(a_{i,j}+\varepsilon_{i,j}^2)+\frac{1}{2}\sum_{j=1}^{n_i-1}L_{i,j}^2+\sum_{j=1}^{n_i}\frac{c_\theta}{2\sigma_{i,j}}\theta_{i,j}^2+B_in_i。$$

3 稳定性分析

定理1：针对式(1)所示的互联非线性时滞系统，在满足假设1和假设2的前提下，基于极限学习机设计的虚拟控制律(见式(18)、式(36))、自适应律(见式(19)、式(37)、式(51))和实际控制器(见式(50))，可以保证系统所有闭环信号有界，跟踪误差收敛到原点附近的小邻域内。

证明：对于整个非线性互联系统，选择如下李雅普诺夫函数

$$V\leqslant\sum_{i=1}^{N}V_{i,n_i}。$$

对 V 求导得：

$$\dot{V}\leqslant\sum_{i=1}^{N}\left[-\sum_{j=1}^{n_i}c_{i,j}z_{i,j}^2-\sum_{j=1}^{n_i}(\frac{1}{l_{i,j}}-1)\chi_{i,j}^2-\sum_{j=1}^{n_i}\frac{c_\theta}{2\sigma_{i,j}}\tilde{\theta}_{i,j}^2-\sum_{j=1}^{n_i}\tau(1-\bar{d}_{i,j})H_{i,j}+\sum_{j=1}^{n_i}(1-k_{i,j}z_{i,j}^2)\Omega_{i,j}+\rho\right],$$

式中 $\rho=\frac{1}{2}\sum_{j=1}^{n_i}(a_{i,j}^2+\varepsilon_{i,j}^2)+\frac{1}{2}\sum_{j=1}^{n_i-1}L_{i,j}^2+\sum_{j=1}^{n_i}\frac{c_\theta}{2\sigma_{i,j}}\theta_{i,j}^2+B_in_i$。

基于以上描述，考虑如下2种情况。

情况1：$(|z_{i,j}|\leqslant\sqrt{1/k_{i,j}},j=1,2,\cdots,n_i)$ 从上面表述中可以看出，由于 $z_{i,1}$、$y_{i,r}$ 的有界性，可以从式(4)得到 $x_{i,1}$ 有界。然后，因为 $\tilde{\theta}_{i,1}$ 是有界的，所以 $\alpha_{i,1}$ 有界。如果 $\alpha_{i,1}$ 有界，可以从式(20)中得到 $\omega_{i,2}$ 也是有界的。基于 $z_{i,2}$、$\omega_{i,2}$ 的有界性，可以得到 $x_{i,2}$ 也是有界的。因为 $\tilde{\theta}_{i,2}$ 有界，那么 $\alpha_{i,2}$ 也有界。从类似的推导中，最终可以得到 $\alpha_{i,j}(j=3,4,\cdots,n_i-1)$ 和 u_i 也是有界的。因此，通过以上分析可以总结得出系统所有闭环信号都是有界的。

情况2：$|z_{i,j}|>\sqrt{1/k_{i,j}}$ 并且 $\Omega_{i,j}\geqslant 0$，所以 $\sum_{j=1}^{n_i}(1-k_{i,j}z_{i,j}^2)\Omega_{i,j}\leqslant 0$ 此时 $\dot{V}$ 可以简写成 $\dot{V}\leqslant-\gamma V+\rho$，其中 $\gamma=\min\limits_{1\leqslant j\leqslant n_i}\left\{2c_{i,j},\bar{\tau}_{i,j},c_\theta,2(\frac{1}{l_{i,j}}-1)\right\}$，$\bar{\tau}_{i,j}=\tau(1-\bar{d}_{i,j})$。故可以得到 $\dot{V}<0$，即系统闭环信号都是有界的。然后，对 $\dot{V}$ 两边同时乘以 $e^{\gamma t}$ 并对不等式的两边再进行积分可以得到 $V(t)\leqslant e^{-\gamma t}V(0)+(\rho/\gamma)(1-e^{-\gamma t})$，根据 $(1/2)z_{i,1}^2\leqslant V(t)$，可以得到 $|z_{i,1}|\leqslant 2\sqrt{\rho/\gamma}$。因此，跟踪误差 $z_{i,1}$ 可以收敛到原点的可调邻域内。

基于极限学习机的分散式自适应跟踪控制系统结构示意图如图1所示。

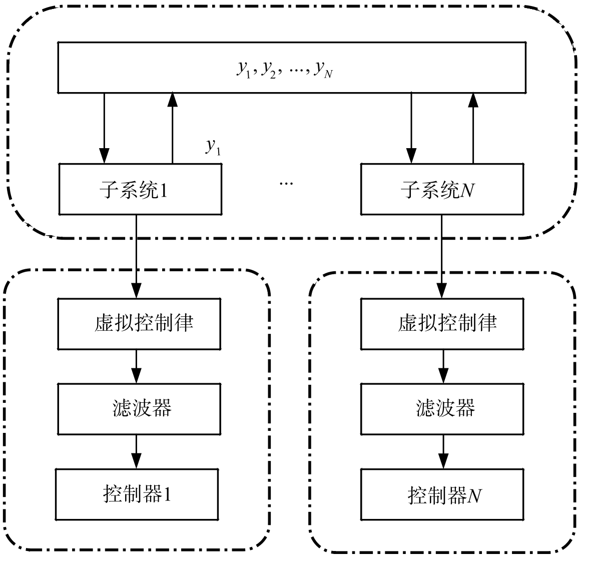

图 1　分散式自适应跟踪控制系统结构示意图

Fig. 1　Schematic diagram of a distributed adaptive tracking control structure

4　仿真验证

为进一步验证所提控制策略的有效性，针对两级化学反应釜系统进行仿真验证。实际系统模型描述如下：

$$\begin{cases}\dot{x}_{1,1}=-\dfrac{1-A_{1,2}}{\mathrm{vol}_{1,1}}x_{1,2}-\dfrac{1}{T_{1,1}}x_{1,1}-K_{1,1}x_{1,1}+C_{1,1}\sin(x_{1,1}+x_{2,1}),\\ \dot{x}_{1,2}=-\dfrac{B_1}{\mathrm{vol}_{1,2}}u_1-\dfrac{1}{T_{1,2}}x_{1,2}-K_{1,2}x_{1,2}-\dfrac{A_{1,1}}{\mathrm{vol}_{1,2}}x_{1,1}(t-d_{1,2})+C_{1,2}\sin(x_{2,1}),\\ y_1=x_{1,1},\end{cases}$$

$$\begin{cases}\dot{x}_{2,1}=-\dfrac{1-A_{2,2}}{\mathrm{vol}_{2,1}}x_{2,2}-\dfrac{1}{T_{2,1}}x_{2,1}-K_{2,1}x_{2,1}+C_{2,1}\sin(x_{1,1}),\\ \dot{x}_{2,2}=-\dfrac{B_2}{\mathrm{vol}_{2,2}}u_2-\dfrac{1}{T_{2,2}}x_{2,2}-K_{2,2}x_{2,2}-\dfrac{A_{2,1}}{\mathrm{vol}_{2,2}}x_{2,1}(t-d_{2,2})+C_{2,2}\sin(x_{1,1}-x_{2,1}),\\ y_2=x_{2,1},\end{cases}$$

式中：$x_{1,1}$、$x_{1,2}$、$x_{2,1}$ 和 $x_{2,2}$ 为各子系统的状态向量；u_1、u_2 为控制输入；$A_{1,1}$、$A_{1,2}$、$A_{2,1}$ 和 $A_{2,2}$ 为化学反应系统的循环流速；$\mathrm{vol}_{1,1}$、$\mathrm{vol}_{1,2}$、$\mathrm{vol}_{2,1}$ 和 $\mathrm{vol}_{2,2}$ 为化学反应系统的体积；$T_{1,1}$、$T_{1,2}$、$T_{2,1}$ 和 $T_{2,2}$ 为化学反应系统的反应堆停留时间；$K_{1,1}$、$K_{1,2}$、$K_{2,1}$ 和 $K_{2,2}$ 为化学反应系统的反应常数；B_1 和 B_2 为化学反应系统的进料速度。

时滞选择为 $d_{1,2}=0.2+0.08\sin(2t)$ 和 $d_{2,2}=0.3+0.12\sin(2t)$。期望轨迹选择 $y_{d,1}=y_{d,2}=0.2+0.5\sin(0.5t)$。$a_{1,1}$、$a_{1,2}$、$a_{2,1}$、$a_{2,2}$ 和 c_θ 是自适应参数权重更新算法中的设计参数；$c_{1,1}$、$c_{1,2}$、$c_{2,1}$ 和 $c_{2,2}$ 是虚拟控制律以及实际控制器中的设计参数；$l_{1,2}$ 和 $l_{2,2}$ 是一阶低通滤波器中的设计参数。

具体的系统参数选择如表 1 所示。

表 1　系统参数

Tab. 1　System parameters

参数	取值	参数	取值	参数	取值	参数	取值
$A_{1,1}$	0.5	$A_{1,2}$	0.5	$A_{2,1}$	0.5	$A_{2,2}$	0.5
$\mathrm{vol}_{1,1}$	0.5	$\mathrm{vol}_{1,2}$	0.5	$\mathrm{vol}_{2,1}$	0.4	$\mathrm{vol}_{2,2}$	0.4
$T_{1,1}$	50	$T_{1,2}$	50	$T_{2,1}$	50	$T_{2,2}$	50
$K_{1,1}$	0.03	$K_{1,2}$	0.03	$K_{2,1}$	0.04	$K_{2,2}$	0.04
$C_{1,1}$	0.1	$C_{1,2}$	0.2	$C_{2,1}$	0.1	$C_{2,2}$	0.2
B_1	5	B_2	5				

仿真中的设计参数选择如表2所示。

表2 设计参数

Tab.2 Design parameters

参数	取值	参数	取值	参数	取值	参数	取值
$x_{1,1}(0)$	0.2	$x_{1,2}(0)$	0.2	$x_{2,1}(0)$	0.8	$x_{2,2}(0)$	0.8
$\hat{\theta}_{1,1}(0)$	1	$\hat{\theta}_{1,2}(0)$	0.5	$\hat{\theta}_{2,1}(0)$	0.5	$\hat{\theta}_{2,2}(0)$	0.4
$a_{1,1}$	5	$a_{1,2}$	5	$a_{2,1}$	2	$a_{2,2}$	5
$c_{1,1}$	10	$c_{1,2}$	20	$c_{2,1}$	200	$c_{2,2}$	1
$l_{1,2}$	0.01	$l_{2,2}$	1				

得到的仿真结果如图2—图9所示。图2和图6给出了子系统1中输出信号 y_1 以及子系统2中输出信号 y_2 跟踪相应参考信号的轨迹，从图中可以看出 y_1、y_2 与参考信号 $y_{d,1}$、$y_{d,2}$ 轨迹非常吻合。为了进一步展示系统跟踪性能，图3和图7给出了子系统1和子系统2中的跟踪误差，从图中可以看出系统具有响应速度快且超调量极小等优点。综上所述，所提出的控制策略能够使化学反应系统的输出信号成功地跟踪参考信号。图4和图8是子系统1的控制输入 u_1 和子系统2的控制输入 u_2，图5和图9是子系统1中的自适应参数 $\hat{\theta}_{1,1}$、$\hat{\theta}_{1,2}$ 和子系统2中的自适应参数 $\hat{\theta}_{2,1}$、$\hat{\theta}_{2,2}$ 的变化过程：可以看出所有信号都是有界的，进一步验证了所提控制策略的有效性。

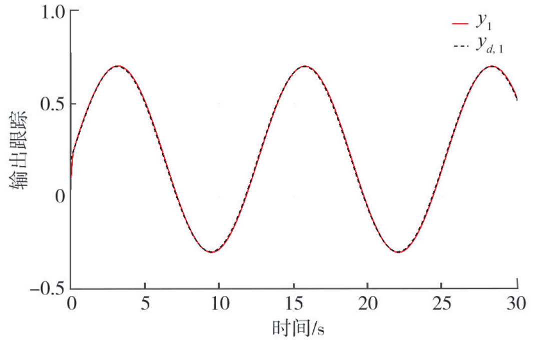

图2 子系统1的输出信号 y_1 跟踪参考信号 $y_{d,1}$ 的轨迹

Fig.2 Output signal y_1 of subsystem 1 in tracking the trajectory of the reference signal $y_{d,1}$

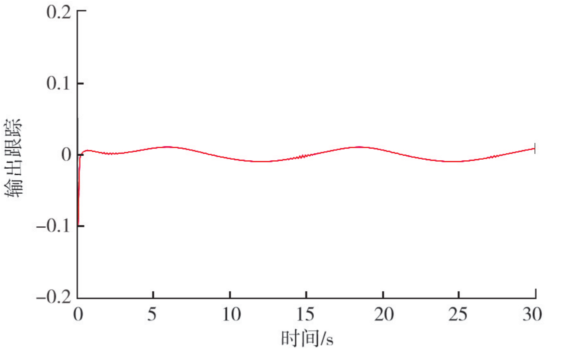

图3 子系统1的跟踪误差 $z_{1,1}$

Fig.3 Tracking error $z_{1,1}$ of subsystem 1

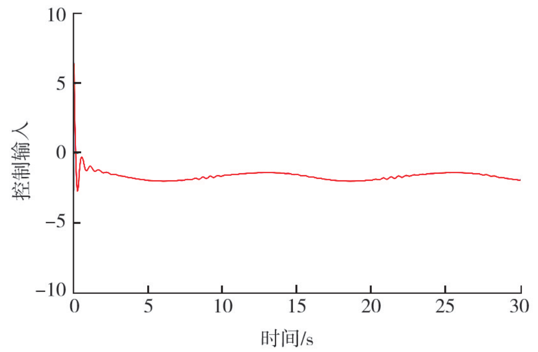

图4 子系统1的控制输入 u_1

Fig.4 Control input u_1 of subsystem 1

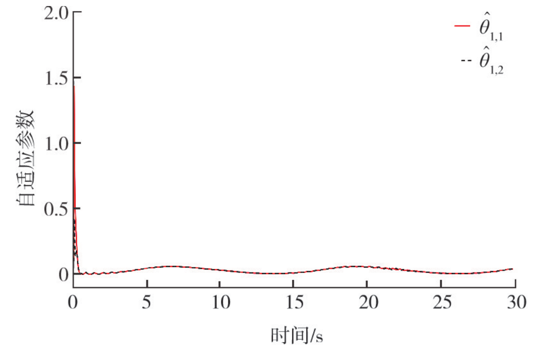

图5 子系统1中的自适应参数 $\hat{\theta}_{1,1}$、$\hat{\theta}_{1,2}$

Fig.5 Adaptive parameters $\hat{\theta}_{1,1}$ and $\hat{\theta}_{1,2}$ in subsystem 1

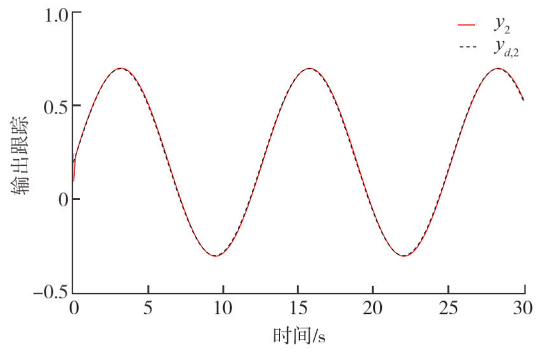

图 6　子系统 2 的输出信号 y_2 跟踪参考信号 $y_{d,2}$ 的轨迹

Fig. 6　Output signal y_2 of subsystem 2 in tracking the trajectory of the reference signal $y_{d,2}$

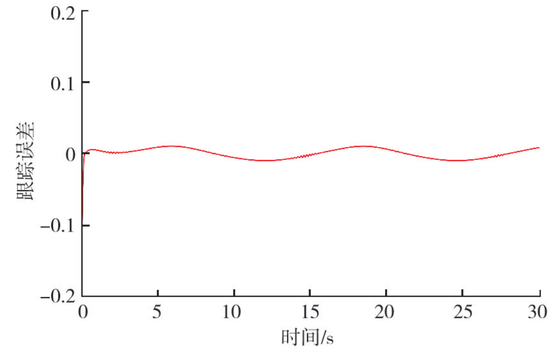

图 7　子系统 2 的跟踪误差 $z_{2,1}$

Fig. 7　Tracking error $z_{2,1}$ of subsystem 2

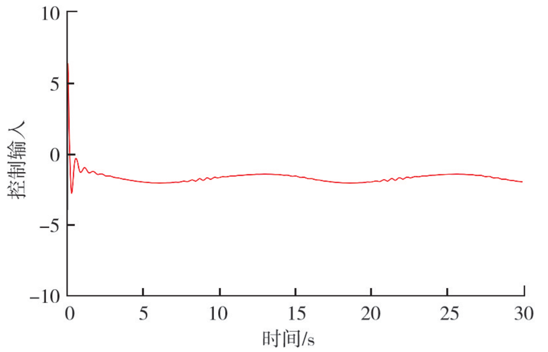

图 8　子系统 2 的控制输入 u_2

Fig. 8　Control input u_2 of subsystem 2

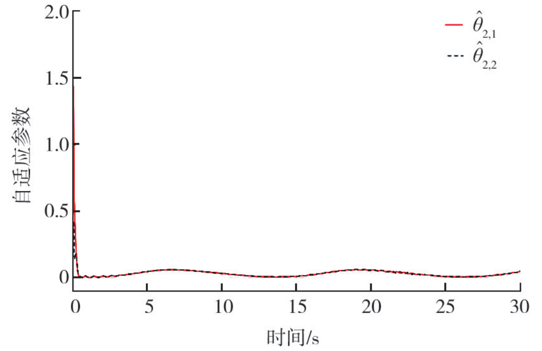

图 9　子系统 2 的自适应参数 $\hat{\theta}_{2,1}$、$\hat{\theta}_{2,2}$

Fig. 9　Adaptive parameters $\hat{\theta}_{2,1}$ and $\hat{\theta}_{2,2}$ in subsystem 2

5　结　语

针对一类互联非线性时滞系统，利用极限学习机的近似特性逼近未知非线性函数，通过 Lyapunov-Krasovskii 函数消除了时滞对互联非线性系统稳定性的影响，结合反演控制技术和动态面控制技术，提出了分散式自适应跟踪控制策略，保证了闭环系统跟踪误差一致最终有界稳定。仿真实例验证了所提控制策略具有明显的有效性和优越性，不仅克服了未知非线性和未知时滞对系统的影响，而且保证了系统具有良好的跟踪性能。未来将进一步考虑执行器故障情况对系统的影响，提出更适合实际应用的容错控制方法，以期为工业自动化、机器人协同控制、航空航天等实际应用领域提供理论参考。

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