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带 p-Laplacian 算子的分数阶微分方程的正解

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摘　要:研究一类带 p-Laplacian 算子的高阶多点 Caputo 分数阶微分方程:

$$\begin{cases} D_{0+}^{\beta}(\phi_p(D_{0+}^{\alpha}u(t)))+f(t,u(t))=0, \quad 0\leqslant t\leqslant 1, l-1<\beta\leqslant l, n-1<\alpha\leqslant n, \\ (\phi_p(D_{0+}^{\alpha}u(0)))^{(i)}=0, \quad i=0,1,2,\cdots,l-1, \\ u^{(i)}(0)=0, \quad i=1,2,\cdots,n-1, \quad u(1)=\sum_{i=1}^{m-2}a_iu(\xi_i)。 \end{cases}$$

运用 Schauder 不动点定理,得到边值问题正解的存在性,最后给出了例子来验证所得结论。

关键词:常微分方程其他学科;p-Laplacian 算子;分数阶微分方程;多点;正解;不动点定理

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Positive solutions for fractional differential equation with a p-Laplacian operator

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Abstract: This paper studies the following Caputo fractional differential equation with p-Laplacian of higher-order multi-point:

$$\begin{cases} D_{0+}^{\beta}(\phi_p(D_{0+}^{\alpha}u(t)))+f(t,u(t))=0, \quad 0\leqslant t\leqslant 1, l-1<\beta\leqslant l, n-1<\alpha\leqslant n, \\ (\phi_p(D_{0+}^{\alpha}u(0)))^{(i)}=0, \quad i=0,1,2,\cdots,l-1, \\ u^{(i)}(0)=0, \quad i=1,2,\cdots,n-1, \quad u(1)=\sum_{i=1}^{m-2}a_iu(\xi_i)。 \end{cases}$$

Using the Schauder fixed point theorem, the existence of positive solution is obtained for the above boundary value problems. An example is presented to illustrate our main theorem.

Keywords: ordinary differential equation; p-Laplacian; fractional differential equation; multi-point; positive solution; fixed point theorem

近年来,分数阶微分方程和带 p-Laplacian 算子的微分方程成为许多数学工作者的研究热点[1-18],在文献[3]中,作者讨论了微分方程:

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$$\begin{cases}(\phi_p(u'(t)))'+a(t)f(t,u,u')=0, & 0\leqslant t\leqslant 1,\\ u'(0)=0, \quad u(1)=\sum\limits_{i=1}^{m-2}\alpha_i u(\xi_i)\end{cases}$$

正解的存在性，其中 $\alpha_i\geqslant 0,(i=1,2,\cdots,m-2)$ 且 $0\leqslant\sum\limits_{i=1}^{m-2}\alpha_i<1$，$f:[0,1]\times[0,+\infty)\times R\to[0,+\infty)$ 是连续的。利用文献[4]中的不动点定理，作者得到上述问题正解的存在性。在文献[5]中，作者讨论了分数阶微分方程：

$$\begin{cases}D_{0+}^{\beta}(\phi_p(D_{0+}^{\alpha}u(t)))=f(t,u(t)), & 0\leqslant t\leqslant 1,\\ u(0)=-u(1), \quad D_{0+}^{\alpha}u(0)=-D_{0+}^{\alpha}u(1)\end{cases}$$

正解的存在性，其中 $f:[0,1]\times[0,+\infty)\to[0,+\infty)$ 是连续的，$0<\alpha\leqslant 1,0<\beta\leqslant 1,1<\alpha+\beta\leqslant 2$，$D_{0+}^{\beta},D_{0+}^{\alpha}$ 是Caputo分数阶微分，利用 Schaefer's 不动点定理，得到上述问题解的存在性。

受此启发，本文来讨论下面高阶多点边值问题的正解情况，

$$\begin{cases}D_{0+}^{\beta}(\phi_p(D_{0+}^{\alpha}u(t)))+f(t,u(t))=0, & 0\leqslant t\leqslant 1,l-1<\beta\leqslant l,n-1<\alpha\leqslant n,\\ (\phi_p(D_{0+}^{\alpha}u(0))^{(i)}=0, & i=0,1,2,\cdots,l-1,\\ u^{(i)}(0)=0, & i=1,2,\cdots,n-1, \quad u(1)=\sum\limits_{i=1}^{m-2}a_i u(\xi_i),\end{cases}\tag{1}$$

其中 $l\geqslant 1,n\geqslant 1,l+n-1<\alpha+\beta\leqslant l+n,D_{0+}^{\beta},D_{0+}^{\alpha}$ 是 Caputo 分数阶微分，$f:[0,1]\times[0,\infty)\to[0,\infty)$ 是连续的，$a_i\geqslant 0,(i=1,2,\cdots,m-2)$ 且 $0\leqslant\sum\limits_{i=1}^{m-2}a_i<1,0=\xi_0<\xi_1<\cdots<\xi_{m-2}<\xi_{m-1}=1$。运用 Schauder 不动点定理，得到该问题正解的存在性。

1 预备知识

为证明结论，需要利用下面的预备知识[19-21]。

定义 1 设函数 $y:(0,\infty)\to R$ 的 $\alpha>0$ 阶 Riemann-Liouville 积分定义如下：

$$I_{0+}^{\alpha}y(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}y(s)\mathrm{d}s,$$

其中右边是在$(0,\infty)$上逐点定义的。

定义 2 设函数 $y:(0,\infty)\to R$ 的 $\alpha>0$ 阶 Caputo 微分定义如下：

$$D_{0+}^{\alpha}y(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t(t-s)^{n-a-1}y^{(n)}(s)\mathrm{d}s,$$

其中 $n=[\alpha]+1$，右边是在$(0,\infty)$上逐点定义的。

引理 1 令 $\alpha>0$，设 $u(t),D_{0+}^{\alpha}u(t)\in L(0,1)$，则：

$$I_{0+}^{\alpha}D_{0+}^{\alpha}u(t)=u(t)+c_0+c_1t+c_2t^2+\cdots+c_{N-1}t^{N-1}, \quad c_i\in R,i=1,2,\cdots,N,$$

其中 N 是大于或等于 α 的最小整数。

引理 2 （Schauder 不动点定理）设 E 是实 Banach 空间，D 是 E 中有界凸闭集（D 不一定有内点），$A:D\to D$ 全连续，则 A 在 D 中必有不动点。

2 主要结果

引理 3 边值问题(1)等价于：

$$u(t)=-\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}r(s)\mathrm{d}s+d_0,\tag{2}$$

其中：

$$r(s)=\phi_q\left(\frac{1}{\Gamma(\beta)}\int_0^s(s-\tau)^{\beta-1}f(\tau,u(\tau))\mathrm{d}\tau\right),\tag{3}$$

$$d_0=\frac{\int_0^1(1-s)^{\alpha-1}r(s)\mathrm{d}s-\sum\limits_{i=1}^{m-2}a_i\int_0^{\xi_i}(\xi_i-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)(1-\sum\limits_{i=1}^{m-2}a_i)},\tag{4}$$

ϕ_q 是 ϕ_p 的逆算子，且 $\frac{1}{p}+\frac{1}{q}=1$。

证明　利用式(1)中的 $D_{0+}^{\beta}(\phi_p(D_{0+}^{\alpha}u(t)))+f(t,u(t))=0$，可得 $D_{0+}^{\beta}(\phi_p(D_{0+}^{\alpha}u(t)))=-f(t,u(t))$。两边从 0 到 t 积分，利用引理 1 得：

$$\phi_p(D_{0+}^{\alpha}u(t))=-\frac{1}{\Gamma(\beta)}\int_0^t(t-\tau)^{\beta-1}f(\tau,u(\tau))\mathrm{d}\tau+c_0+c_1t+\cdots+c_{l-1}t^{l-1},$$

再利用式(1)中的 $(\phi_p(D_{0+}^{\alpha}u(0))^{(i)}=0,i=0,1,2,\cdots,l-1$，可得 $c_0=c_1=\cdots=c_{l-1}=0$，所以

$$\phi_p(D_{0+}^{\alpha}u(t))=-\frac{1}{\Gamma(\beta)}\int_0^t(t-\tau)^{\beta-1}f(\tau,u(\tau))\mathrm{d}\tau,$$

两边作用 ϕ_q，得 $D_{0+}^{\alpha}u(t)=-\phi_q\left(\frac{1}{\Gamma(\beta)}\int_0^t(t-\tau)^{\beta-1}f(\tau,u(\tau))\mathrm{d}\tau\right)$。

对上式两边从 0 到 t 积分，利用引理 1 得：

$$u(t)=-\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}\phi_q\left(\frac{1}{\Gamma(\beta)}\int_0^s(s-\tau)^{\beta-1}f(\tau,u(\tau))\mathrm{d}\tau\right)\mathrm{d}s+d_0+d_1t+\cdots+d_{n-1}t^{n-1}=$$

$$-\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}r(s)\mathrm{d}s+d_0+d_1t+\cdots+d_{n-1}t^{n-1},$$

利用式(1)中的 $u^{(i)}(0)=0,i=1,\cdots,n-1$，得 $d_1=\cdots=d_{n-1}=0$，所以 $u(t)=-\frac{\int_0^t(t-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)}+d_0$，

再利用式(1)中的 $u(1)=\sum_{i=1}^{m-2}a_iu(\xi_i)$，可得 $d_0=\frac{\int_0^1(1-s)^{\alpha-1}r(s)\mathrm{d}s-\sum_{i=1}^{m-2}a_i\int_0^{\xi_i}(\xi_i-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)(1-\sum_{i=1}^{m-2}a_i)}$。

证明完毕。

引理 4　式(2)中的 $u(t)$ 是单调不增的，且 $u(t)\geqslant 0$。

证明　因为 $r(s)=\phi_q\left(\frac{1}{\Gamma(\beta)}\int_0^s(s-\tau)^{\beta-1}f(\tau,u(\tau))\mathrm{d}\tau\right)$，而 $f(\tau,u(\tau))\geqslant 0$，所以 $r(s)\geqslant 0$。又因为 $u(t)=-\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}r(s)\mathrm{d}s+d_0$，所以 $u'(t)=-\frac{1}{\Gamma(\alpha-1)}\int_0^t(t-s)^{\alpha-2}r(s)\mathrm{d}s\leqslant 0$。因此 $u(t)$ 关于 t 是单调不增的。

因为 $u(t)$ 是单调不增的，所以 $u(t)\geqslant u(1),\forall\, t\in[0,1]$。而

$$u(1)=-\frac{\int_0^1(1-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)}+\frac{\int_0^1(1-s)^{\alpha-1}r(s)\mathrm{d}s-\sum_{i=1}^{m-2}a_i\int_0^{\xi_i}(\xi_i-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)(1-\sum_{i=1}^{m-2}a_i)}=$$

$$\frac{\sum_{i=1}^{m-2}a_i\int_0^1(1-s)^{\alpha-1}r(s)\mathrm{d}s-\sum_{i=1}^{m-2}a_i\int_0^{\xi_i}(\xi_i-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)(1-\sum_{i=1}^{m-2}a_i)}\geqslant$$

$$\frac{\sum_{i=1}^{m-2}a_i\int_{\xi_i}^1(1-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)(1-\sum_{i=1}^{m-2}a_i)}\geqslant 0,$$

所以 $u(t)\geqslant 0$。证明完毕。

令 $E=C[0,1]$，$\forall\, u\in C[0,1]$，定义 $\|u\|=\max_{0\leqslant t\leqslant 1}|u|$，因此 E 是 Banach 空间。定义算子 $T:E\to E$ 为 $Tu(t)=-\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}r(s)\mathrm{d}s+d_0$，其中 $r(s),d_0$ 如式(3)和式(4)所示。

定理 1　令 $f:[0,1]\times[0,\infty)\to[0,\infty)$ 是连续的。设存在非负函数 $a(t),b(t)\in E$ 使得：

$$f(t,u)\leqslant a(t)u^{p-1}+b(t),\quad \forall\, t\in[0,1],u\in[0,\infty)。$$

若 $\Gamma(\beta+1)[\Gamma(\alpha+1)(1-\sum_{i=1}^{m-2}a_i)]^{p-1}\geqslant\|a\|$，则边值问题(1)至少有1个正解。

证明 证明分为3步。

第1步，令 $B=\{u\in E:|u|\leqslant l,\forall t\in[0,1]\}$，其中：

$$l=\frac{\|b\|^{q-1}}{\{\Gamma(\beta+1)[\Gamma(\alpha+1)(1-\sum_{i=1}^{m-2}a_i)]^{p-1}-\|a\|\}^{q-1}},$$

显然 $B\subset E$ 是一非空有界闭凸集。$\forall u\in B$，有：

$$r(s)=\phi_q\left(\frac{1}{\Gamma(\beta)}\int_0^s(s-\tau)^{\beta-1}f(\tau,u(\tau))\mathrm{d}\tau\right)\leqslant$$

$$\phi_q\left(\frac{1}{\Gamma(\beta)}\int_0^1(1-\tau)^{\beta-1}(\|a\|l^{p-1}+\|b\|)\mathrm{d}\tau\right)=$$

$$\phi_q\left(\frac{(\|a\|l^{p-1}+\|b\|)}{\Gamma(\beta+1)}\right)=\left(\frac{(\|a\|l^{p-1}+\|b\|)}{\Gamma(\beta+1)}\right)^{q-1},$$

而

$$Tu(t)=-\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}r(s)\mathrm{d}s+d_0\leqslant$$

$$d_0=\frac{\int_0^1(1-s)^{\alpha-1}r(s)\mathrm{d}s-\sum_{i=1}^{m-2}a_i\int_0^{\xi_i}(\xi_i-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)(1-\sum_{i=1}^{m-2}a_i)}\leqslant$$

$$\frac{\int_0^1(1-s)^{\alpha-1}r(s)\mathrm{d}s}{\Gamma(\alpha)(1-\sum_{i=1}^{m-2}a_i)}\leqslant$$

$$\frac{\left(\frac{(\|a\|l^{p-1}+\|b\|)}{\Gamma(\beta+1)}\right)^{q-1}\int_0^1(1-s)^{\alpha-1}\mathrm{d}s}{\Gamma(\alpha)(1-\sum_{i=1}^{m-2}a_i)}=$$

$$\frac{\left(\frac{(\|a\|l^{p-1}+\|b\|)}{\Gamma(\beta+1)}\right)^{q-1}}{\Gamma(\alpha+1)(1-\sum_{i=1}^{m-2}a_i)}=l,$$

因此 $T:B\to B$。

第2步，由于 $f:[0,1]\times[0,\infty)\to[0,\infty)$ 是连续的，所以 T 是连续算子。

第3步，令 $0\leqslant t_1\leqslant t_2\leqslant 1$，$M_0=\sup_{t\in[0,1],u\in B}f(t,u)$，则对任意的 $u\in B$，有：

$$r(s)=\phi_q\left(\frac{1}{\Gamma(\beta)}\int_0^s(s-\tau)^{\beta-1}f(\tau,u(\tau))\mathrm{d}\tau\right)\leqslant\phi_q\left(\frac{M_0}{\Gamma(\beta)}\int_0^1(1-\tau)^{\beta-1}\mathrm{d}\tau\right)=\left(\frac{M_0}{\Gamma(\beta+1)}\right)^{q-1}\triangleq L_0,$$

所以

$$|Tu(t_1)-Tu(t_2)|=\left|\frac{1}{\Gamma(\alpha)}\int_0^{t_2}(t_2-s)^{\alpha-1}r(s)\mathrm{d}s-\frac{1}{\Gamma(\alpha)}\int_0^{t_1}(t_1-s)^{\alpha-1}r(s)\mathrm{d}s\right|\leqslant$$

$$\left|\frac{1}{\Gamma(\alpha)}\int_{t_1}^{t_2}(t_2-s)^{\alpha-1}r(s)\mathrm{d}s\right|+\left|\frac{1}{\Gamma(\alpha)}\int_0^{t_1}((t_2-s)^{\alpha-1}-(t_1-s)^{\alpha-1})r(s)\mathrm{d}s\right|\leqslant$$

$$\frac{L_0}{\Gamma(\alpha)}\left|\int_{t_1}^{t_2}(t_2-s)^{\alpha-1}\mathrm{d}s\right|+\frac{L_0}{\Gamma(\alpha)}\left|\int_0^{t_1}((t_2-s)^{\alpha-1}-(t_1-s)^{\alpha-1})\mathrm{d}s\right|=$$

$$\frac{L_0}{\Gamma(\alpha+1)}(t_2-t_1)^{\alpha}+\frac{L_0}{\Gamma(\alpha+1)}(t_2^{\alpha}-t_1^{\alpha}-(t_2-t_1)^{\alpha})=\frac{L_0}{\Gamma(\alpha+1)}(t_2^{\alpha}-t_1^{\alpha})。$$

综合上面的结论，利用 Arzela-Ascoli 定理，得 $T:B\to B$ 是全连续算子。运用 Schauder 不动点定理，得算子 T 至少有1个不动点，即边值问题(1)至少有1个解。又因为 $u(t)\geqslant 0$，所以边值问题(1)至少有1个正解。证明完毕。

3　举　例

讨论下面边值问题的正解情况：

$$\begin{cases} D_{0+}^{2.5}(\phi_2(D_{0+}^{1.6}u(t))) + (\sin t)u(t) + t^2 = 0, \quad 0 \leqslant t \leqslant 1, \\ (\phi_2(D_{0+}^{1.6}u(0)))^{(i)} = 0, \quad i = 0,1, \\ u^{(i)}(0) = 0, \quad i = 1,2, \quad u(1) = 0.1u(0.2) + 0.3u(0.5), \end{cases}$$

显然

$$f(t,u) = (\sin t)u(t) + t^2 \leqslant tu(t) + t^2, \quad \forall\, t \in [0,1], u \in [0,\infty),$$

$$\Gamma(2.5+1)\Gamma(1.6+1)(1-0.1-0.3) \approx 2.85 \geqslant \| t \| = 1,$$

利用定理 1，上述边值问题至少有 1 个正解。

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