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一类细胞神经网络的时滞相关全局稳定性判据

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摘 要: 研究了一类带有离散和分布时间滞后的不确定时滞细胞神经网络(DCNN)的全局渐进稳定性。应用李亚普诺夫稳定性理论, 构造李亚普诺夫泛函, 结合 Leibniz-Newton 公式, 给出一个关于时滞细胞神经网络的新颖的全局渐进稳定性判据, 所得出的结论依赖于时间滞后的最大值并且以线性矩阵不等式的形式给出。最后给出一个数值例子来说明所提判据的有效性和可行性。

关键词: 时滞相关; 细胞神经网络; 线性矩阵不等式(LMIs); 全局渐进稳定性; 离散和分布式的时滞

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Delay-dependent global asymptotic stability criteria for a class of cellular neural networks

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Abstract: The global asymptotic stability for a class of uncertain delayed cellular neural networks (DCNN) with discrete and distributed time-varying delays is studied in this paper. By using Lyapunov stability theory to construct a Lyapunov-Krasovskii function, and combining Leibniz-Newton formula, we get a novel global asymptotic stability criteria for DCNN. The final results are proposed via LMI approach. An numerical example is given to illustrate the effectiveness and feasibility of our proposed approach.

Key words: delay-dependent; cellular neural networks; linear matrix inequalities(LMIs); global asymptotic stability; discrete and distributed time-varying delays

近年来关于神经网络的研究已经成为许多科技工作者关注的问题, 其研究成果已经广泛应用于各种信息处理系统, 包括相关成分的探测、固定点的计算、孔的填充、图像的影像、最优化、联合记忆、模式分类和信号处理等[1,2]。为了解决移动的图像处理, 文献[3]引入了时滞细胞神经网络的概念和模型。由于电子网络中放大器的有限转化速度或者生物网络中的信号传播的有限速度, 使得神经网络中出现时滞是很自然的现象, 因此研究时滞细胞神经网络的全局渐进稳定性具有重要的理论意义。近年来, 一些学者研究离散和分布式时滞人工神经网络的稳定性[4~6]。考虑到在大规模的集成芯片运行过程中, 参数的摄动将破坏细胞神经网络的稳定性, 所以本文将研究带离散和分布式时滞的不确定细胞神经网络的全局渐进稳定性。

根据稳定性判据本身是否依赖于时滞大小, 目前所得到的稳定性判据可以分为 2 类, 一类是时滞相关型

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判据[5,7~9]; 一类是时滞无关型判据[4,6,10,11]。通常前者在时滞小的时候, 保守性比较小。在基于李亚普诺夫函数的时滞相关型判据中, 时滞慢变化率条件 $\dot{h}(t) < 1$ 通常拘泥于离散式的变时滞[7~10]。在文献[7, 10]中, 代数稳定性判据是通过 Halanay 不等式提出的。在文献[8]中, 时滞相关型判据是通过控制假象轴没有特征值的 Hamiltonian 矩阵得到的。用代数判据或 Hamiltonian 矩阵求解是比较困难的。应用 LMIs 方法处理许多控制问题是很有效的, 并且易用 Matlab 的工具箱进行仿真[12,13]。在本文中, 通过李亚普诺夫稳定性理论, 构造适当的李亚普诺夫泛函, 结合 Leibniz-Newton 公式, 改进了文献[14]中时滞相关型判据, 所得出的结论比现有的文献结论具有较小的保守性。

1　问题的描述

考虑下面 n 维时滞细胞神经网络系统:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{W}_0 f(\boldsymbol{x}(t)) + \boldsymbol{W}_1 f(\boldsymbol{x}(t-h(t))) + \boldsymbol{W}_2\int_{t-\tau(t)}^{t} f(\boldsymbol{x}(s))\mathrm{d}s + \boldsymbol{J},\quad t \geqslant 0, \tag{1a}$$

$$\boldsymbol{x}(t) = \phi(t),\quad t \in [-H, 0], \tag{1b}$$

其中: $\boldsymbol{x}(t) = [x_1(t)x_2(t)\cdots x_n(t)]^{\mathrm{T}}$; $f(\boldsymbol{x}(t)) = [f_1(x_1)f_2(x_2)\cdots f_n(x_n)]^{\mathrm{T}}$, 是有界的神经元激活函数, 并且 $f_i(0) = 0$; 有界函数 $h(t)$ 和 $\tau(t)$ 分别代表着所考虑的神经网络的离散滞后和分布式滞后, 并且满足 $0 \leqslant h(t) \leqslant h_M$, $\dot{h}(t) \leqslant h_D$, $0 \leqslant \tau(t) \leqslant \tau_M$; $H = \max\{h_M, \tau_M\}$; $\boldsymbol{J} = [J_1 J_2 \cdots J_n]^{\mathrm{T}}$, 是外部斜线向量; $\boldsymbol{A} = \mathrm{diag}[a_i]$, $a_i > 0$; $\boldsymbol{W}_i \in \mathbf{R}^{n\times n}$, $i = 0, 1, 2$, 是常数矩阵, 并且初始向量 $\phi \in \boldsymbol{C}_1$。激活函数 $f_i(x_i)$, $i = 0, 1, 2, \cdots, n$ 是有界单调不增的, 满足

$$0 \leqslant \frac{f_i(\xi_1) - f_i(\xi_2)}{\xi_1 - \xi_2} \leqslant \eta_i,\quad \xi_1, \xi_2 \in \mathbf{R},\quad \xi_1 \neq \xi_2,\quad i = 1, 2, \cdots, n, \tag{2}$$

式中 $\eta_i > 0$, $i = 1, 2, \cdots, n$ 是给定的常数。

假设 $\boldsymbol{x} = [x_1 x_2 \cdots x_n]^{\mathrm{T}} \in \mathbf{R}$ 是系统(1)的平衡点, 那么可以得到下面的系统:

$$\dot{\boldsymbol{z}}(t) = -\boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{W}_0 g(\boldsymbol{z}(t)) + \boldsymbol{W}_1 g(\boldsymbol{z}(t-h(t))) + \boldsymbol{W}_2\int_{t-\tau(t)}^{t} g(z(s))\mathrm{d}s, \tag{3}$$

式中: $\boldsymbol{z}(t) = [z_1(t)z_2(t)\cdots z_n(t)]^{\mathrm{T}} = \boldsymbol{x}(t) - \boldsymbol{x}$; $\boldsymbol{g}(\boldsymbol{z}(t)) = [g_1(z_1(t))g_2(z_2(t))\cdots g_n(z_n(t))]^{\mathrm{T}}$,

$$g_i(z_i(t)) = f_i(x_i(t)) - f_i(x_i) = f_i(z_i(t) + x_i) - f_i(x_i),\ g_i(0) = 0, \tag{4a}$$

$$g_i(z_i(t-h(t))) = f_i(x_i(t-h(t))) - f_i(x_i) = f_i(z_i(t-h(t)) + x_i) - f_i(x_i)。 \tag{4b}$$

由式(2)和式(4a)、式(4b), 有

$$0 \leqslant \frac{g_i(\xi_1) - g_i(\xi_2)}{\xi_1 - \xi_2} = \frac{f_i(\xi_1 + x_i) - f_i(x_i) - [f_i(\xi_2 + x_i) - f_i(x_i)]}{(\xi_1 + \tilde{x}_i) - (\xi_2 + \tilde{x}_i)} \leqslant \eta_i,\ \xi_1 \neq \xi_2, \tag{4c}$$

$$|g_i(z_i(t))| = |f_i(z_i(t) + x_i) - f_i(x_i)| \leqslant \eta_i |z_i(t)|,\quad i = 1, 2, \cdots, n。 \tag{4d}$$

2　预备知识

引理 1[11]　对于一个给定的矩阵 $\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{12}^{\mathrm{T}} & \boldsymbol{S}_{22} \end{bmatrix}$, 其中 $\boldsymbol{S}_{11} = \boldsymbol{S}_{11}^{\mathrm{T}}$, $\boldsymbol{S}_{22} = \boldsymbol{S}_{22}^{\mathrm{T}}$, 则下面的 2 个条件是等价的: 1) $\boldsymbol{S} < 0$;　2) $\boldsymbol{S}_{22} < 0$,　$\boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}} < 0$。

引理 2[12]　假定 $\boldsymbol{U}, \boldsymbol{V}, \boldsymbol{W}$ 和 $\boldsymbol{X}$ 是适当维数的实矩阵, 其中 $\boldsymbol{X}$ 满足 $\boldsymbol{X} = \boldsymbol{X}^{\mathrm{T}}$, 则:

$$\boldsymbol{X} + \boldsymbol{U}\boldsymbol{V}\boldsymbol{W} + \boldsymbol{W}^{\mathrm{T}}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{U}^{\mathrm{T}} < 0,\ \forall \boldsymbol{V}^{\mathrm{T}}\boldsymbol{V} \leqslant \boldsymbol{I},$$

等价于: 存在一个正标量 $\varepsilon > 0$, 下面的矩阵不等式成立:

$$\boldsymbol{X} + \varepsilon^{-1}\boldsymbol{U}\boldsymbol{U}^{\mathrm{T}} + \varepsilon\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W} = \boldsymbol{X} + \varepsilon^{-1}\boldsymbol{U}\boldsymbol{U}^{\mathrm{T}} + \varepsilon^{-1}(\varepsilon\boldsymbol{W})^{\mathrm{T}}(\varepsilon\boldsymbol{W}) < 0。$$

3　主要结论

下面给出系统(1)的时滞相关的全局渐近稳定性判据。

定理 1　系统(1)的平衡点 $\boldsymbol{x}$ 对于 $h_D < 1$ 是全局渐近稳定的, 如果存在着正定对称矩 $\boldsymbol{P}_0$, $\boldsymbol{P}_1$, $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, 正标量 $\alpha > 0$, 使得下面的矩阵不等式成立。

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$$
\boldsymbol{\Xi}=\begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & -\boldsymbol{N}_1\\
* & \Xi_{22} & -\boldsymbol{M}_2-\boldsymbol{N}_2^{\mathrm T} & \boldsymbol{M}_2\boldsymbol{W}_0-\boldsymbol{N}_4^{\mathrm T} & \boldsymbol{M}_2\boldsymbol{W}_1-\boldsymbol{N}_5^{\mathrm T} & \boldsymbol{M}_2\boldsymbol{W}_2-\boldsymbol{N}_6^{\mathrm T} & -\boldsymbol{N}_2\\
* & * & \Xi_{33} & \boldsymbol{M}_3\boldsymbol{W}_0-\boldsymbol{M}_4^{\mathrm T} & \boldsymbol{M}_3\boldsymbol{W}_1-\boldsymbol{M}_5^{\mathrm T} & \boldsymbol{M}_3\boldsymbol{W}_2 & -\boldsymbol{N}_3\\
* & * & * & \Xi_{44} & \boldsymbol{M}_4\boldsymbol{W}_1 & \boldsymbol{M}_4\boldsymbol{W}_2 & -\boldsymbol{N}_4\\
* & * & * & * & \Xi_{55} & \boldsymbol{M}_5\boldsymbol{W}_2 & -\boldsymbol{N}_5\\
* & * & * & * & * & \Xi_{66} & -\boldsymbol{N}_6\\
* & * & * & * & * & * & -\boldsymbol{R}_1
\end{bmatrix}<0, \tag{5}
$$

式中:

$\Xi_{11}=-\boldsymbol{P}_0\boldsymbol{A}-\boldsymbol{A}^{\mathrm T}\boldsymbol{P}_0+\boldsymbol{P}_1+\boldsymbol{N}_1+\boldsymbol{N}_1^{\mathrm T}-\boldsymbol{M}_1\boldsymbol{A}-\boldsymbol{A}^{\mathrm T}\boldsymbol{M}_1^{\mathrm T}$; $\Xi_{12}=-\boldsymbol{N}_1-\boldsymbol{A}^{\mathrm T}\boldsymbol{M}_2^{\mathrm T}+\boldsymbol{N}_2^{\mathrm T}$; $\Xi_{13}=-\boldsymbol{M}_1-\boldsymbol{A}^{\mathrm T}\boldsymbol{M}_3^{\mathrm T}+\boldsymbol{N}_3^{\mathrm T}$; $\Xi_{14}=\boldsymbol{P}_0\boldsymbol{W}_0+\boldsymbol{M}_1\boldsymbol{W}_0+\alpha\boldsymbol{L}^{\mathrm T}-\boldsymbol{A}^{\mathrm T}\boldsymbol{M}_4^{\mathrm T}+\boldsymbol{N}_4^{\mathrm T}$; $\Xi_{15}=\boldsymbol{P}_0\boldsymbol{W}_1+\boldsymbol{M}_1\boldsymbol{W}_1-\boldsymbol{A}^{\mathrm T}\boldsymbol{M}_5^{\mathrm T}+\boldsymbol{N}_5^{\mathrm T}$; $\Xi_{16}=\boldsymbol{P}_0\boldsymbol{W}_2+\boldsymbol{M}_1\boldsymbol{W}_2+\boldsymbol{N}_6^{\mathrm T}$; $\Xi_{22}=-(1-h_D)\boldsymbol{P}_1-\boldsymbol{N}_2-\boldsymbol{N}_2^{\mathrm T}$; $\Xi_{33}=h_M^2\boldsymbol{R}_1-\boldsymbol{M}_3-\boldsymbol{M}_3^{\mathrm T}$; $\Xi_{44}=\boldsymbol{P}_2+\tau_M^2\boldsymbol{R}_2+\boldsymbol{M}_4\boldsymbol{W}_0+\boldsymbol{W}_0^{\mathrm T}\boldsymbol{M}_4^{\mathrm T}-2\alpha\boldsymbol{I}$; $\Xi_{55}=-(1-h_D)\boldsymbol{P}_2+\boldsymbol{M}_5\boldsymbol{W}_1+\boldsymbol{W}_1^{\mathrm T}\boldsymbol{M}_5^{\mathrm T}$; $\Xi_{66}=-\boldsymbol{R}_2+\boldsymbol{M}_6\boldsymbol{W}_2+\boldsymbol{W}_2^{\mathrm T}\boldsymbol{M}_6^{\mathrm T}$。

证明 构造李亚普诺夫函数

$$
V(\boldsymbol{z}(t))=V_1(\boldsymbol{z}(t))+V_2(\boldsymbol{z}(t))+V_3(\boldsymbol{z}(t))+V_4(\boldsymbol{z}(t))+V_5(\boldsymbol{z}(t)), \tag{6}
$$

式中:

$V_1(\boldsymbol{z}(t))=\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{P}_0\boldsymbol{z}(t)$; $V_2(\boldsymbol{z}(t))=\int_{t-h(t)}^{t}\boldsymbol{z}^{\mathrm T}(s)\boldsymbol{P}_1\boldsymbol{z}(s)\mathrm{d}s$; $V_3(\boldsymbol{z}(t))=\int_{t-h(t)}^{t}\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(s))\boldsymbol{P}_2\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s$;

$V_4(\boldsymbol{z}(t))=h_M\int_{-h_M}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{z}}^{\mathrm T}(s)\boldsymbol{R}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta$; $V_5(\boldsymbol{z}(t))=\tau_M\int_{-\tau_M}^{0}\int_{t+\theta}^{t}\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(s))\boldsymbol{R}_1\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s\mathrm{d}\theta$。

沿着系统(3)的轨迹的 $V(\boldsymbol{z}(t))$ 对时间的导数满足

$$
\begin{aligned}
\dot V(\boldsymbol{z}(t))={}&\boldsymbol{z}^{\mathrm T}(t)(-\boldsymbol{P}_0\boldsymbol{A}-\boldsymbol{A}^{\mathrm T}\boldsymbol{P}_0)\boldsymbol{z}(t)+2\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{P}_0\boldsymbol{W}_0\boldsymbol{g}(\boldsymbol{z}(t))+2\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{P}_0\boldsymbol{W}_1\boldsymbol{g}(\boldsymbol{z}(t-h(t)))+\\
&2\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{P}_0\boldsymbol{W}_2\int_{t-\tau(t)}^{t}\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s+\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{P}_1\boldsymbol{z}(t)-(1-\dot h(t))\boldsymbol{z}^{\mathrm T}(t-h(t))\boldsymbol{P}_1\boldsymbol{z}(t-h(t))+\\
&\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t))\boldsymbol{P}_1\boldsymbol{g}(\boldsymbol{z}(t))-(1-\dot h(t))\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t-h(t)))\boldsymbol{P}_1\boldsymbol{g}(\boldsymbol{z}(t-h(t)))+h_M^2\dot{\boldsymbol{z}}^{\mathrm T}(t)\boldsymbol{R}_1\dot{\boldsymbol{z}}(t)-\\
&h_M\int_{t-h_M}^{t}\dot{\boldsymbol{z}}^{\mathrm T}(s)\boldsymbol{R}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s+\tau_M^2\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t)\boldsymbol{R}_2\boldsymbol{g}(\boldsymbol{z}(t)))-\tau_M\int_{t-\tau_M}^{t}\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(s))\boldsymbol{R}_2\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s,
\end{aligned}\tag{7}
$$

利用 Jensen 不等式,有

$$
-h_M\int_{t-h_M}^{t}\dot{\boldsymbol{z}}^{\mathrm T}(s)\boldsymbol{R}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s\leqslant-h(t)\int_{t-h(t)}^{t}\dot{\boldsymbol{z}}^{\mathrm T}(s)\boldsymbol{R}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s\leqslant-\left[\int_{t-h(t)}^{t}\dot{\boldsymbol{z}}(s)\mathrm{d}s\right]^{\mathrm T}\boldsymbol{R}_1\left[\int_{t-h(t)}^{t}\dot{\boldsymbol{z}}(s)\mathrm{d}s\right], \tag{8}
$$

$$
\begin{aligned}
-\tau_M\int_{t-h_M}^{t}\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(s))\boldsymbol{R}_2\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s&\leqslant-h(t)\int_{t-\tau(t)}^{t}\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(s))\boldsymbol{R}_2\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s\leqslant\\
&-\left[\int_{t-\tau(t)}^{t}\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s\right]^{\mathrm T}\boldsymbol{R}_2\left[\int_{t-\tau(t)}^{t}\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s\right],
\end{aligned}\tag{9}
$$

利用 Leibniz-Newton 公式,有

$$
2\boldsymbol{\xi}^{\mathrm T}(t)\boldsymbol{N}\left[\boldsymbol{z}(t)-\boldsymbol{z}(t-h(t))-\int_{t-h(t)}^{t}\dot{\boldsymbol{z}}(s)\mathrm{d}s\right]=0, \tag{10}
$$

$$
2\boldsymbol{\xi}^{\mathrm T}\boldsymbol{M}\left[-\dot{\boldsymbol{z}}(t)-\boldsymbol{A}\boldsymbol{z}(t)+\boldsymbol{W}_0\boldsymbol{g}(\boldsymbol{z}(t))+\boldsymbol{W}_1\boldsymbol{g}(\boldsymbol{z}(t-h(t)))+\boldsymbol{W}_2\int_{t-\tau(t)}^{t}\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s\right]=0, \tag{11}
$$

其中:

$$
\boldsymbol{\xi}^{\mathrm T}(t)=\left[\boldsymbol{z}^{\mathrm T}(t)\quad \boldsymbol{z}^{\mathrm T}(t-h(t))\quad \dot{\boldsymbol{z}}^{\mathrm T}(t)\quad \boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t))\quad \boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t-h(t)))\quad \int_{t-\tau(t)}^{t}\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(s))\mathrm{d}s\right];
$$

$$
\boldsymbol{N}=[\boldsymbol{N}_1^{\mathrm T}\quad \boldsymbol{N}_2^{\mathrm T}\quad \boldsymbol{N}_3^{\mathrm T}\quad \boldsymbol{N}_4^{\mathrm T}\quad \boldsymbol{N}_5^{\mathrm T}\quad \boldsymbol{N}_6^{\mathrm T}]^{\mathrm T},\quad \boldsymbol{M}=[\boldsymbol{M}_1^{\mathrm T}\quad \boldsymbol{M}_2^{\mathrm T}\quad \boldsymbol{M}_3^{\mathrm T}\quad \boldsymbol{M}_4^{\mathrm T}\quad \boldsymbol{M}_5^{\mathrm T}\quad \boldsymbol{M}_6^{\mathrm T}]^{\mathrm T}。
$$

由式(4d),易知:

$$
2\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t))\boldsymbol{g}(\boldsymbol{z}(t))\leqslant 2\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t))\boldsymbol{L}\boldsymbol{z}(t)。 \tag{12}
$$

从式(7)-式(12),可得如下不等式:

$$
\begin{aligned}
&\dot V(\boldsymbol{z}(t))+2\boldsymbol{\xi}^{\mathrm T}\boldsymbol{M}\left[-\dot{\boldsymbol{z}}(t)-\boldsymbol{A}\boldsymbol{z}(t)+\boldsymbol{W}_0\boldsymbol{g}(\boldsymbol{z}(t))+\boldsymbol{W}_1\boldsymbol{g}(\boldsymbol{z}(t-h(t)))+\boldsymbol{W}_2\int_{t-\tau(t)}^{t}\boldsymbol{g}(\boldsymbol{z}(s))\mathrm{d}s\right]+\\
&2\boldsymbol{\xi}^{\mathrm T}(t)\boldsymbol{N}\left[\boldsymbol{z}(t)-\boldsymbol{z}(t-h(t))-\int_{t-h(t)}^{t}\dot{\boldsymbol{z}}(s)\mathrm{d}s\right]+2\alpha\left[\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t))\boldsymbol{L}\boldsymbol{z}(t)-\boldsymbol{g}^{\mathrm T}(\boldsymbol{z}(t))\boldsymbol{g}(\boldsymbol{z}(t))\right]\leqslant\boldsymbol{\eta}^{\mathrm T}(t)\boldsymbol{\Xi}\boldsymbol{\eta}(t),
\end{aligned}
$$

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其中$\eta^{T}(t)=[z^{T}(t)\quad z^{T}(t-h(t))\quad \dot{z}^{T}(t)\quad g^{T}(z(t))\quad g^{T}(z(t-h(t)))\quad \int_{t-\tau(t)}^{t} g^{T}(z(s))\mathrm{d}s\quad \int_{t-h(t)}^{t}\dot{z}^{T}(s)\mathrm{d}s]$，$\Xi$在式(5)中给出。因为$\Xi<0$，所以存在一个常数$\beta>0$，使得$\dot{V}(z(t))\leqslant-\beta\|z(t)\|^{2}$，故系统(1)的平衡点$x$是全局渐进稳定的。定理证毕。

基于定理 1 的证明，笔者将其推广到参数不确定时滞细胞神经网络的情况，首先考虑如下模型：

$$\dot{x}(t)=-[A+\Delta A(t)]x(t)+[W_0+\Delta W_0(t)]g(x(t))+[W_1+\Delta W_1(t)]g(x(t-h(t)))+[W_2+\Delta W_2(t)]\int_{t-\tau(t)}^{t} g(x(s))\mathrm{d}s+J。\tag{13}$$

其中系统的变量和部分参数在系统(1)中已经给出，$\Delta A(t)$，$\Delta W_0(t)$，$\Delta W_1(t)$，$\Delta W_2(t)$是时变的不确定性矩阵，并且满足如下形式：

$$[\Delta A(t)\quad \Delta W_0(t)\quad \Delta W_1(t)\quad \Delta W_2(t)]=YF(t)[S_A\quad S_0\quad S_1\quad S_2]。\tag{14}$$

式中：M，S_A，S_i，$i\in\{0,1,2\}$是已知的适当维数的常矩阵；$F(t)$是未知的时变函数且满足如下条件：

$$F^{T}(t)F(t)\leqslant I。$$

定理 2　系统(13)的平衡点x对于$h_D<1$是全局渐进稳定的，如果存在着正定对称矩阵P_0，P_1，P_2，R_1，R_2，正标量$\alpha>0$和$\varepsilon>0$，使得下面的矩阵不等式成立：

$$\begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & -N_1 & P_0Y+M_1Y & -\varepsilon\cdot S_A^{T}\\
* & \Xi_{22} & -M_2-N_2^{T} & M_2W_0-N_4^{T} & M_2W_1-N_5^{T} & M_2W_2-N_6^{T} & -N_2 & M_2Y & 0\\
* & * & \Xi_{33} & M_3W_0-M_4^{T} & M_3W_1-M_5^{T} & M_3W_2 & -N_3 & M_3Y & 0\\
* & * & * & \Xi_{44} & M_4W_1 & M_4W_2 & -N_4 & M_4Y & \varepsilon\cdot S_0^{T}\\
* & * & * & * & \Xi_{55} & M_5W_2 & -N_5 & M_5Y & \varepsilon\cdot S_1^{T}\\
* & * & * & * & * & \Xi_{66} & -N_6 & M_6Y & \varepsilon\cdot S_2^{T}\\
* & * & * & * & * & * & -R_1 & 0 & 0\\
* & * & * & * & * & * & * & -\varepsilon\cdot I & 0\\
* & * & * & * & * & * & * & * & -\varepsilon\cdot I
\end{bmatrix}<0。\tag{15}$$

证明　对式(1)应用引理 1，可知矩阵不等式(15)等价于如下不等式：

$$\Xi+\varepsilon^{-1}\begin{bmatrix}P_0Y+M_1Y\\ M_2Y\\ M_3Y\\ M_4Y\\ M_5Y\\ M_6Y\\ 0\end{bmatrix}\begin{bmatrix}P_0Y+M_1Y\\ M_2Y\\ M_3Y\\ M_4Y\\ M_5Y\\ M_6Y\\ 0\end{bmatrix}^{T}+\varepsilon\begin{bmatrix}-S_A^{T}\\ 0\\ 0\\ S_0^{T}\\ S_1^{T}\\ S_2^{T}\\ 0\end{bmatrix}^{T}\begin{bmatrix}-S_A^{T}\\ 0\\ 0\\ S_0^{T}\\ S_1^{T}\\ S_2^{T}\\ 0\end{bmatrix}<0。$$

在式(5)的矩阵Ξ中，用$A+\Delta A(t)$，$W_0+\Delta W_0(t)$，$W_1+\Delta W_1(t)$，$W_2+\Delta W_2(t)$分别替换A，W_0，W_1，W_2，经过计算并且应用引理 2，能够得到如下矩阵不等式：

$$\Xi+\begin{bmatrix}P_0Y+M_1Y\\ M_2Y\\ M_3Y\\ M_4Y\\ M_5Y\\ M_6Y\\ 0\end{bmatrix}F(t)\begin{bmatrix}-S_A^{T}\\ 0\\ 0\\ S_0^{T}\\ S_1^{T}\\ S_2^{T}\\ 0\end{bmatrix}^{T}+\begin{bmatrix}-S_A^{T}\\ 0\\ 0\\ S_0^{T}\\ S_1^{T}\\ S_2^{T}\\ 0\end{bmatrix}F^{T}(t)\begin{bmatrix}P_0Y+M_1Y\\ M_2Y\\ M_3Y\\ M_4Y\\ M_5Y\\ M_6Y\\ 0\end{bmatrix}^{T}<0。$$

从而由定理 1 知，不确定时滞神经网络系统(13)是全局渐进稳定的。

4　数值例子

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下面,举出一个例子来证明所提出的方法的有效性和可行性。

例:考虑文献[14]中所列举的时滞细胞神经网络控制系统(1),其参数矩阵如下:

$$A=\begin{pmatrix}2.3&0&0\\0&3.4&0\\0&0&2.5\end{pmatrix},\ W_0=\begin{pmatrix}0.9&-1.5&0.1\\-1.2&1&0.2\\0.2&0.3&0.8\end{pmatrix},$$

$$W_1=\begin{pmatrix}0.8&0.6&0.2\\0.5&0.7&0.1\\0.2&0.1&0.5\end{pmatrix},\ W_2=\begin{pmatrix}0.3&0.2&0.1\\0.1&0.2&0.1\\0.1&0.1&0.2\end{pmatrix}。$$

$L=0.2I$,通过应用Matlab的LMI工具箱求解定理1可知,所考虑的时滞细胞神经网络控制系统(1)是全局渐进稳定的,同时解得系统所允许的最大时滞$h_M=\tau_M=25.71$,显然笔者所得出的最大时滞比文献[14]中所得出的最大时滞$h_M=\tau_M=23.7$要大,这说明所得出的时滞相关性稳定性判据具有较低的保守性。所解得定理1中的未知矩阵和标量如下:

$$P_0=10^3\begin{pmatrix}1.9979&-0.1447&-0.6279\\-0.1447&2.0257&-0.0414\\-0.6279&-0.0414&2.5913\end{pmatrix},\quad P_1=10^3\begin{pmatrix}0.4858&-0.5355&-0.4956\\-0.5355&2.8301&-0.7582\\-0.4956&-0.7582&1.2655\end{pmatrix},$$

$$P_3=10^3\begin{pmatrix}6.1247&-5.0440&-3.7663\\-5.0440&7.6230&3.3476\\-3.7663&3.3476&4.1731\end{pmatrix},\quad \alpha=10^4\times 9.7237,$$

$$R_1=\begin{pmatrix}0.4284&-0.4039&-0.3202\\-0.4039&0.5494&-0.0050\\-0.3202&-0.0050&0.8240\end{pmatrix},\quad R_2=\begin{pmatrix}112.7527&18.4383&9.9445\\18.4383&95.6020&62.8885\\9.9445&62.8885&63.4875\end{pmatrix},$$

$$M_1=10^4\begin{pmatrix}0.2492&-5.9806&-2.0619\\8.5690&0.1474&4.6134\\2.3949&-3.4490&0.1556\end{pmatrix},\quad M_2=\begin{pmatrix}0.0689&-0.0760&-0.0845\\-0.0343&0.1084&-0.0033\\-0.0691&-0.0108&0.2319\end{pmatrix},$$

$$M_3=10^4\begin{pmatrix}0.2894&-2.6627&-0.9144\\2.4420&0.1562&1.3807\\0.9298&-1.3701&0.2962\end{pmatrix},\quad M_4=10^4\begin{pmatrix}3.9852&2.0750&2.6883\\-3.8933&-2.5581&-2.7837\\-0.6081&1.2472&1.4585\end{pmatrix},$$

$$M_5=10^4\begin{pmatrix}-1.6221&2.3383&0.0138\\-1.9386&1.6653&-0.4285\\-0.7676&1.1789&-0.0460\end{pmatrix},\quad M_6=10^3\begin{pmatrix}-4.1241&9.2382&1.0822\\-6.3227&6.4655&-1.1932\\-4.5803&5.2821&-0.9131\end{pmatrix},$$

$$N_1=\begin{pmatrix}0.1788&-0.2321&-0.0210\\-0.1947&0.2233&0.0739\\0.0356&-0.0598&0.0208\end{pmatrix},\quad N_2=\begin{pmatrix}0.3963&-0.3772&-0.2896\\-0.3424&0.4522&0.0210\\-0.3030&0.0071&0.7577\end{pmatrix},$$

$$N_3=\begin{pmatrix}0.3072&-0.3457&-0.1282\\-0.2391&0.2829&0.0754\\-0.1295&-0.0286&0.3822\end{pmatrix},\quad N_4=\begin{pmatrix}0.2062&0.3604&0.1525\\-0.4777&0.4890&0.2938\\-0.1074&-0.4067&0.4888\end{pmatrix},$$

$$N_5=\begin{pmatrix}1.9979&-0.1447&-0.6279\\-0.1447&2.0257&-0.0414\\-0.6279&-0.0414&2.5913\end{pmatrix},\quad N_6=\begin{pmatrix}-0.0603&0.0827&-0.0030\\-0.0015&0.0158&-0.0263\\0.0211&0.0109&-0.0742\end{pmatrix}。$$

5 结 论

本文考虑了带离散和分布式时间滞后的不确定神经网络控制系统的全局渐进稳定性,通过构造适当的Lyapunov-Krasovskii泛函,应用Leibniz-Newton公式,以线性矩阵不等式的形式给出了能够保证所考虑神经网络控制系统全局渐进稳定性的时滞相关型判据。最后,举出一个数值例子说明所提判据的有效性和较低的保守性。

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