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Sturm-Liouville

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摘　要:为了丰富Sturm-Liouville(S-L)微分算子的谱理论,研究了闭区间[0,1]上边界条件依赖谱参数的非连续S-L问题。首先利用该问题在直和空间上的等价刻画,给出了非连续S-L问题特征值与连续S-L问题特征值间的交替关系,即在非连续S-L问题的特征值的每个开子区间内都恰有连续S-L问题的一个特征值,进而由连续S-L问题的振荡理论推出非连续S-L问题的振荡理论。然后通过Prüfer变换和Hergloz函数的转换,建立了边界条件依赖谱参数的非连续S-L问题与边界条件为常值的非连续S-L问题的转换,得出转换后的特征值与转换前(除去有限个)的特征值相等。最后通过构造边界条件为常值的非连续S-L问题的特征函数求得其特征值的渐近式,从而得到了边界条件依赖谱参数的非连续S-L问题的特征值的渐近表达式。新的研究方法可推广到对间断点条件依赖谱参数的S-L问题研究。

关键词:算子代数;Sturm-Liouville微分算子;非连续条件;参数边界条件

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Spectral distribution problem for Sturm-Liouville with operators the discontinuity conditions at an interior point and boundary conditions depending on the eigenparameter

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Abstract: In order to enrich the spectral theory of Sturm-Liouvillel (S-L) differential operators, the discontinuous S-L problem with boundary conditions dependent on spectral parameters on closed interval [0,1] is studied. Firstly, by using the equivalent characterization of the problem in the direct sum space, the alternating relation between the eigenvalues of the discontinuous S-L problem and the eigenvalues of the continuous S-L problem is given. That is, there is exactly one eigenvalue of the continuous S-L problem in every open subinterval of the eigenvalues of the discontinuous S-L problem, and then the oscillation theory of the discontinuous S-L problem is derived from the oscillation theory of the continuous S-L problem. Through the transforma-

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tions of Prüfer and Hergloz function, the transformation between the discontinuous S-L problem with boundary conditions dependent spectral parameters and discontinuous S-L problem with constant boundary conditions is established. The obtained converted eigenvalues are equal to those (excluding the finite eigenvalues) before the conversion. Finally, the asymptotic expressions of eigenvalues of discontinuous S-L problems with boundary conditions dependent on spectral parameters are obtained by constructing the eigenfunctions of discontinuous S-L problems with constant boundary conditions. The new research method can be extended to the study of the S-L problem with boundary conditions dependent spectral parameters.

Keywords: operator algebras; Sturm-Liouville differential operator; discontinuity conditions; eigenparameter-dependent boundary condition

Sturm-Liouville(S-L) ,
。 , S-L , :
[1-4]、 [5-7]、 [8-10] [11-13];
, [14-16] S-L , ,
“ ” “ ”, 。

S-L :

$$ly := -y'' + q(x)y = \lambda y, \quad x \in [0,1] \backslash \{\frac{1}{2}\}, \tag{1}$$

:

$$U(y) := y(0)\cos\alpha - y'(0)\sin\alpha = 0, \quad \alpha \in [0,\pi), \tag{2}$$

$$V(y) := (\frac{y'}{y})(1) - f(\lambda) = 0 \tag{3}$$

:

$$P(y) := y(\frac{1}{2}-) - y(\frac{1}{2}+) = 0, \tag{4}$$

$$Q(y) := y'(\frac{1}{2}-) - y'(\frac{1}{2}+) + by(\frac{1}{2}-) = 0 \tag{5}$$

。 $\lambda \in \mathbf{C}$, $q(x) \in L^2[0,1]$,

$$f(\lambda) = a\lambda + b - \sum_{k=1}^{N} \frac{b_k}{\lambda - c_k}, \tag{6}$$

$a, b, b_k, c_k \in \mathbf{R}$ $a \geqslant 0, b_k \geqslant 0, c_1 < c_2 < \cdots < c_N, N \geqslant 0$。 , (1)— (5) ,
、 [17-18]。 , $b=0$, S-L (1)— (3)。 (1)—
(3) (1)— (5) $\{\eta_n\}_{n=0}^{+\infty}$ $\{\lambda_n\}_{n=0}^{+\infty}$。

S-L (1)— (5) , , $q(x) \equiv 0$
S-L $\{\lambda_n^0\}_{n=0}^{+\infty}$, $\{\lambda_n^0\}_{n=0}^{+\infty}$ [19-21]。
, S-L S-L 。 , [1]
S-L , $\{\lambda_n^0\}_{n=0}^{+\infty}$ 。

1 预备知识

S-L , 。

Herglotz :

$$R_N = \{f : f(\lambda) = a\lambda + b - \sum_{k=1}^{N} \frac{b_k}{\lambda - c_k}, a, b, b_k, c_k \in \mathbf{R}, a \geqslant 0, b_k \geqslant 0, c_1 < c_2 < \cdots < c_N, N \geqslant 0\},$$

R_N^+ $a > 0$ R_N , R_N^0 $a = 0$ R_N 。

$\mu < c_1$, :

$$F(\lambda)=\frac{\mu-\lambda}{f(\lambda)-f(\mu)}-f(\mu),\tag{7}$$

， $f\in R_N$， $F\in R_M$， ：

$$F(\lambda)=A\lambda+B-\sum_{k=1}^{M}\frac{B_k}{\lambda-C_k}。\tag{8}$$

$\{\eta_i^D\}_{i=0}^{+\infty}$ S-L ：

$$\begin{cases}-y''+q(x)y=\lambda y,\\ y(0)\cos\alpha-y'(0)\sin\alpha=0,\quad \alpha\in[0,\pi),\\ y(1)=0,\end{cases}$$

$\{\eta_i^{cD}\}_{i=0}^{+\infty}=\{c_j\}_{j=0}^{+\infty}\cup\{\eta_i^D\}_{i=0}^{+\infty}$( ) ， $k_j=\#\{c_i\leqslant\eta_j\}$。

， 。

引理 1[22]

1) $f\in R_N^+$， $F\in R_N^0$ $\mu<c_1<C_1<c_2<\cdots<c_N<C_N$；

2) $f\in R_N^0$， $F\in R_{N-1}^+$ $\mu<c_1<C_1<c_2<\cdots<C_{N-1}<c_N$。

注 1 R_N^+ R_N^0， R_N^+ S-L S-L

。

引理 2[22]

1) η_i η_i^{cD} ：$\eta_0<\eta_0^{cD}\leqslant\eta_1\leqslant\eta_1^{cD}\leqslant\cdots$；

2) $c_{k_j}\leqslant\eta_j\leqslant c_{k_j+1}$；

3) $\eta_{-1}^D=-\infty$， $j\geqslant 0$， $\eta_{j-1-k_j}^D<\eta_j\leqslant\eta_{j-k_j}^D$。

引理 3[22] S-L (1)— (3)($f\in R_N$) S-L ：

$$\begin{cases}-y''+q_0y=\mu y,\\ \dfrac{y'}{y}(j)=\cot\alpha_j,\quad j=0,1,\end{cases}$$

$\{\mu_j\}_{j=0}^{+\infty}$， ：

1) $\alpha>0, f\in R_N^+$， $\alpha_0=0,\mu_j=\eta_{j+N+1}$；

2) $\alpha=0, f\in R_N^0$， $\alpha_0=0,\mu_j=\eta_{j+N}$；

3) ，$\alpha_0>0,\mu_j=\eta_{j+N}$。

S-L (1)— (5)

$$H=\{y\in L^2[0,1]:y,y'\in AC[0,1],ly\in L^2[0,1],U(y)=V(y)=P(y)=Q(y)=0\},$$

$$H_{1/2}^-=\{y\in H:y|[\tfrac{1}{2},1]=0\};$$

$$H_{1/2}^+=\{y\in H:y|[0,\tfrac{1}{2}]=0\};$$

$$H_{1/2}=\{y\in H:y(\tfrac{1}{2})=0\},$$

$$S_{1/2}(\lambda)=\{y\in H:y|[0,\tfrac{1}{2}]\in L^2[0,\tfrac{1}{2}],y|[\tfrac{1}{2},1]\in L^2[\tfrac{1}{2},1],$$

$$(l-\lambda)y=0,x\in[0,\tfrac{1}{2})\cup(\tfrac{1}{2},1],U(y)=V(y)=0\}。$$

注 2 $y\in H$， ， $y\in S_{1/2}(\lambda)$， $y(\frac{1}{2}-)=y(\frac{1}{2}+)$， y $x=\frac{1}{2}$ 。

引理 4 $y,\tilde{y}\in L^2[0,1]$，

$$\int_0^1 (ly)\tilde{y}\,\mathrm{d}x=\delta_0(y,\tilde{y})-\delta_1(y,\tilde{y})+\int_0^1 P(y,\tilde{y})\,\mathrm{d}x, \tag{9}$$

:$\delta_x(y,\tilde{y})=y'(x)\tilde{y}(x)$;$P(y,\tilde{y})=y'\tilde{y}'+q(x)y\tilde{y}$。

$y\in S_{1/2}(\lambda),\tilde{y}\in H_{1/2}^{-}$， $\beta_0(y,\tilde{y})$ $\beta_1(y,\tilde{y})$:$\delta_0(y,\tilde{y})=\beta_0(y,\tilde{y}),\delta_1(y,\tilde{y})=\beta_1(y,\tilde{y})$。

$E(y,\tilde{y})=\beta_0(y,\tilde{y})-\beta_1(y,\tilde{y})+\int_0^1 P(y,\tilde{y})\mathrm{d}x$，$E_\lambda(y,\tilde{y})=E(y,\tilde{y})-\lambda\int_0^1(y\tilde{y})\mathrm{d}x$。

注 3 $H_{1/2}^{-}$ $H_{1/2}^{+}$ $E_\lambda(y,\tilde{y})$ ， $H_{1/2}=H_{1/2}^{-}\oplus H_{1/2}^{+}$。

2 主要结论与证明

， (1)— (3) (1)— (5) ，
(1)— (5) 。

定理 1

1)$S_{1/2}(\lambda)$ $E_\lambda(y,\tilde{y})$ $H_{1/2}^{-}$ ；

2) $y(\frac{1}{2}-,\lambda)\neq 0,y(\frac{1}{2}+,\lambda)\neq 0$， $H=H_{1/2}\oplus S_{1/2}(\lambda)$。

证明 1) $S_{1/2}(\lambda)$ $H_{1/2}^{-}$ 。

$y\in S_{1/2}(\lambda),\tilde{y}\in H_{1/2}^{-}$， $\int_0^1(ly)\tilde{y}\mathrm{d}x=\int_0^{1/2}(ly)\tilde{y}\mathrm{d}x$。 4 :

$$\int_0^{1/2}(ly)\tilde{y}\,\mathrm{d}x=\delta_0(y,\tilde{y})-\delta_{1/2}(y,\tilde{y})+\int_0^{1/2}P(y,\tilde{y})\,\mathrm{d}x,$$

$y\in S_{1/2}(\lambda),\tilde{y}\in H_{1/2}^{-}$， ，

$$\delta_0(y,\tilde{y})=\beta_0(y,\tilde{y}),$$

$$\delta_{1/2}(y,\tilde{y})=\beta_{1/2}(y,\tilde{y})=y'(\frac{1}{2})\tilde{y}(\frac{1}{2})=0,$$

$$\int_0^{1/2}P(y,\tilde{y})\,\mathrm{d}x=\int_0^1 P(y,\tilde{y})\,\mathrm{d}x,$$

$\beta_1(y,\tilde{y})=0$， $\int_0^1 P(y,\tilde{y})\mathrm{d}x=E(y,\tilde{y})$。 $E_\lambda(y,\tilde{y})=\int_0^1(l-\lambda)y\tilde{y}\mathrm{d}x=0$， $S_{1/2}(\lambda)$ $H_{1/2}^{-}$
。 ,$S_{1/2}(\lambda)$ $H_{1/2}^{+}$ ， $S_{1/2}(\lambda)$ $H_{1/2}$ 。

$y\in H$ $H_{1/2}^{-}$ 。 1 1) ， $y\in S_{1/2}(\lambda)$。

$$E_\lambda(y,\tilde{y})=0,\quad \forall\,\tilde{y}\in H_{1/2},$$

$y\in L^2[0,\frac{1}{2}]\cup L^2[\frac{1}{2},1]$， $\tilde{y}\in H_{1/2}\cap L^2[0,1]$， ，

$$\begin{aligned}\int_0^1(ly)\tilde{y}\,\mathrm{d}x&=-\delta_0(y,\tilde{y})-\delta_{1/2}(y,\tilde{y})+\int_0^1 P(y,\tilde{y})\,\mathrm{d}x=\\&\delta_0(y,\tilde{y})-\delta_1(y,\tilde{y})+\int_0^1 P(y,\tilde{y})\,\mathrm{d}x=\\&(\delta_0(y,\tilde{y})-\beta_0(y,\tilde{y}))+(\beta_1(y,\tilde{y})-\delta_1(y,\tilde{y}))+E(y,\tilde{y}),\end{aligned}$$

$$\int_0^1(l-\lambda)y\tilde{y}\,\mathrm{d}x=(\delta_0(y,\tilde{y})-\beta_0(y,\tilde{y}))+(\beta_1(y,\tilde{y})-\delta_1(y,\tilde{y})),$$

，

$$(l-\lambda)y=0,\quad \forall x\in[0,\frac{1}{2})\cup(\frac{1}{2},1],$$

$$\delta_0(y,\tilde{y})=\beta_0(y,\tilde{y}),\tag{10}$$

$$\delta_1(y,\tilde{y})=\beta_1(y,\tilde{y})。\tag{11}$$

(10)— (11) y 2) 3)， $y\in S_{1/2}(\lambda)$ 。

2) $\{\lambda_n^-\}_{n=0}^{+\infty}$ $\{\lambda_n^+\}_{n=0}^{+\infty}$ 2 S-L ：

$$\begin{cases}-y''+q(x)y=\lambda y,\\ y(0)\cos\alpha-y'(0)\sin\alpha=0,\quad \alpha\in[0,\pi),\\ y(\frac{1}{2})=0,\end{cases}$$

$$\begin{cases}-y''+q(x)y=\lambda y,\\ y(\frac{1}{2})=0,\\ (\frac{y'}{y})(1)-f(\lambda)=0,\end{cases}$$

$y(\frac{1}{2}-,\lambda)\neq 0, y(\frac{1}{2}+,\lambda)\neq 0$， $\lambda\notin\{\lambda_n^-\}_{n=0}^{+\infty}\cup\{\lambda_n^+\}_{n=0}^{+\infty}$。

$$E_\lambda(y,\tilde{y})=0,\quad \forall\tilde{y}\in H,\tag{12}$$

λ (1)— (5) ， $y\in H(y\neq 0)$ (12)。 $\lambda\notin\{\lambda_n^-\}_{n=0}^{+\infty}\cup\{\lambda_n^+\}_{n=0}^{+\infty}$， $\tilde{y}\in H_{1/2}$ $y\in H_{1/2}(y\neq 0)$ (12)， λ $H_{1/2}$ 。

2)， $y\in H_{1/2}$ $y_0\in H$， $y_h\in H_{1/2}$ $E_\lambda(y_0-y_h,y)=0$。

1) $y_0-y_h=s\in S_{1/2}(\lambda)$， ， $y\in H_{1/2}$， $y_h\in H_{1/2}$ $E_\lambda(y_h,y)=E_\lambda(y_0,y)$。 λ $H_{1/2}$ ， $f:H_{1/2}\to\mathbf{R}$ $y\in H_{1/2}$， $y_h\in H_{1/2}$ $E_\lambda(y_h,y)=f(\lambda)$。 ， $y\in H_{1/2}$ $y_h\in H_{1/2}$， $E_\lambda(y_h,y)=E_\lambda(y_0,y)$， 1 。

$H=H_{1/2}\oplus S_{1/2}(\lambda)$ S-L (1)— (5)。

引理 5[23] $\{\lambda_n\}_{n=0}^{+\infty}$ S-L (1)— (5) ，

$$\{\lambda_n\}_{n=0}^{+\infty}=\{\lambda_n^-\}_{n=0}^{+\infty}\cup\{\lambda_n^+\}_{n=0}^{+\infty}\cup\{\lambda_n^S\}_{n=0}^{+\infty}。$$

引理 6[23] $\{\lambda_n\}_{n=0}^{+\infty}=\{\lambda_n^-\}_{n=0}^{+\infty}\cup\{\lambda_n^+\}_{n=0}^{+\infty}\cup\{\lambda_n^S\}_{n=0}^{+\infty}$ ， $\mathbf{R}$ ，

$$-\infty<\lambda_0<\lambda_1<\cdots<\lambda_n<\cdots,$$

1) $(\lambda_{n-1},\lambda_n)(n=0,1,\cdots)$ S-L (1)— (3) η_n；

2) $\lambda_k\in\{\lambda_n^-\}_{n=0}^{+\infty}\cup\{\lambda_n^+\}_{n=0}^{+\infty}\cup\{\lambda_n^S\}_{n=0}^{+\infty}$， λ_k (1)— (3) 。

5 6 2。

定理 2 S-L (1)— (5) S-L (1)— (3) $\{\lambda_n\}_{n=0}^{+\infty}$ $\{\eta_n\}_{n=0}^{+\infty}$， ：

$$-\infty:=\lambda_{-1}<\eta_0<\lambda_0\leqslant\eta_1\leqslant\lambda_1\leqslant\cdots,\tag{13}$$

， $\lambda_{n-1}=\eta_n=\lambda_n$ 。

注 4 (13) S-L ， (1)— (3) (1)— (5) 。

$\{\lambda_i^D\}_{n=0}^{+\infty}$ S-L ：

$$\begin{cases}-y''+q(x)y=\lambda y,\\ y(0)\cos\alpha-y'(0)\sin\alpha=0,\quad \alpha\in[0,\pi),\\ y(1)=0,\\ y(\frac{1}{2}-)-y(\frac{1}{2}+)=0,\\ y'(\frac{1}{2}-)-y'(\frac{1}{2}+)+by(\frac{1}{2}-)=0。\end{cases}$$

2 ，$\{\eta_i^D\}_{n=0}^{+\infty}$ $\{\lambda_i^D\}_{i=0}^{+\infty}$ ：$\eta_0^D<\lambda_0^D\leqslant\eta_1^D\leqslant\lambda_1^D\leqslant\cdots$， ， $\lambda_{n-1}^D=\eta_n^D=\lambda_n^D$ 。 $\{\lambda_i^{cD}\}_{i=0}^{+\infty}=\{c_j\}_{j=0}^{+\infty}\cup\{\lambda_i^D\}_{i=0}^{+\infty}$（ ） 。 $\tilde{k}_j=\#\{c_i\leqslant\lambda_j\}$， ，$\tilde{k}_j=k_j$。 2 3。

定理 3

1）λ_i λ_i^{cD} ：$\lambda_0<\lambda_0^{cD}\leqslant\lambda_1\leqslant\lambda_1^{cD}\leqslant\cdots$；

2）$c_{\tilde{k}_j}\leqslant\lambda_j\leqslant c_{\tilde{k}_j+1}$；

3） $\lambda_{-1}^D=-\infty$， $j\geqslant 0$， $\lambda_{j-1-\tilde{k}_j}^D<\lambda_j\leqslant\lambda_{j-\tilde{k}_j}^D$。

（1）— （5） 。 ： $\lambda\in(\lambda_{i=1}^D,\lambda_i^D]$， （0，1） （1）— （2） i 。

推论 1 ω_j λ_j ， $\omega_j=j-k_j$， ，$\omega_0=0$， $\lambda_j>c_N$ ，$\omega_j=j-N$。 1 3 4。

定理 4 S-L （1）— （5），$f\in R_N$， S-L ：

$$\begin{cases}-y''+\tilde{q}_0y=\tilde{\mu}y,\\ (\frac{y'}{y})(j)=\cot\tilde{\alpha}_j,\quad j=0,1,\\ y(\frac{1}{2}-)=y(\frac{1}{2}+),\\ y'(\frac{1}{2}-)=y'(\frac{1}{2}+)+by(\frac{1}{2}-),\end{cases}$$

$\{\tilde{\mu}_j\}_{j=0}^{+\infty}$，

1） $\alpha>0,f\in R_N^+$， $\tilde{\alpha}_0=0$， $\tilde{\mu}_j=\lambda_{j+N+1}$；

2） $\alpha=0,f\in R_N^0$， $\tilde{\alpha}_0=0$， $\tilde{\mu}_j=\lambda_{j+N}$；

3） ，$\tilde{\alpha}_0>0,\tilde{\mu}_j=\lambda_{j+N}$。

S-L 。

[0，1] S-L L'：

$$l'y:=-y''+q(x)y=\lambda' y,\tag{14}$$

：

$$U'(y):=y'(0)-hy(0)=0,\quad V'(y):=y'(1)-Hy(1)=0\tag{15}$$

$P(y):=y(\frac{1}{2}-)-y(\frac{1}{2}+)=0,Q(y):=y'(\frac{1}{2}-)-y'(\frac{1}{2}+)-by(\frac{1}{2}-)=0,\lambda'$ ， $q(x)\in L^2[0,1],h,H,b\in\mathbf{R}$。 L' ， 、 。

$y(x)$ $v(x)$ $[0,\frac{1}{2}]$ $[\frac{1}{2},1]$ ， $\langle y,v\rangle:=yv'-y'v$。

$y(x)$ $v(x)$ 4） 5），

$$\langle y,v\rangle_{x=\frac{1}{2}+0}=\langle y,v\rangle_{x=\frac{1}{2}-0},\tag{16}$$

$\langle y,v\rangle$ $[0,1]$ 。 $y(x,\lambda)$ $v(x,\lambda)$ $ly=\lambda y$ $lv=\mu v$ ，

$$\frac{\mathrm{d}}{\mathrm{d}x}\langle y,v\rangle=(\lambda-\mu)yv。\tag{17}$$

$\tau(x,\lambda'),\psi(x,\lambda'),C(x,\lambda'),S(x,\lambda')$ (14) ， ：

$$\tau(0,\lambda')=1,\quad \psi(1,\lambda')=1,\quad C(0,\lambda')=1,\quad S(0,\lambda')=0,$$

$$\tau'(0,\lambda')=h,\quad \psi'(1,\lambda')=-H,\quad C'(0,\lambda')=0,\quad S'(0,\lambda')=1$$

4) 5)， $U'(\tau)=V'(\psi)=0$。

$\Delta(\lambda')=\langle\tau(x,\lambda'),\psi(x,\lambda')\rangle$， (16) Liouville ，$\Delta(\lambda')$ x 。$\Delta(\lambda')$ L' ， ：

$$\Delta(\lambda')=-V'(\tau)=U'(\psi),\tag{18}$$

L' $\{\lambda'_n\}_{n=0}^{+\infty}$ $\Delta(\lambda')$ 。

$C_0(x,\lambda')$ $S_0(x,\lambda')$ $[0,1]$ (14) ， ：

$$C(0,\lambda')=1,\quad S(0,\lambda')=0,$$

$$C'(0,\lambda')=0,\quad S'(0,\lambda')=1,$$

4) 5) ：

$$\begin{cases}C(x,\lambda')=C_0(x,\lambda'),\\ S(x,\lambda')=S_0(x,\lambda'),\end{cases}\quad x<\frac{1}{2},\tag{19}$$

$$\begin{cases}C(x,\lambda')=A_1C_0(x,\lambda')+B_1S_0(x,\lambda'),\\ S(x,\lambda')=A_2C_0(x,\lambda')+B_2S_0(x,\lambda'),\end{cases}\quad x>\frac{1}{2},\tag{20}$$

：

$$\begin{cases}A_1=C_0(\frac{1}{2},\lambda')S'_0(\frac{1}{2},\lambda')-C'_0(\frac{1}{2},\lambda')S_0(\frac{1}{2},\lambda')-bC_0(\frac{1}{2},\lambda')S_0(\frac{1}{2},\lambda'),\\ B_1=bC_0(\frac{1}{2},\lambda'),\\ A_2=bS_0(\frac{1}{2},\lambda'),\\ B_2=C_0(\frac{1}{2},\lambda')S'_0(\frac{1}{2},\lambda')-C'_0(\frac{1}{2},\lambda')S_0(\frac{1}{2},\lambda')+bC_0(\frac{1}{2},\lambda')S_0(\frac{1}{2},\lambda'),\end{cases}\tag{21}$$

$\lambda'=\rho^2,\rho=\sigma+i\zeta$。 S-L $C_0(x,\lambda')$ ：

$$C_0(x,\lambda')=\cos\rho x+\int_0^x\frac{\sin\rho(x-t)}{\rho}q(t)C_0(t,\lambda')\mathrm{d}t,\tag{22}$$

$|\rho|\to\infty$ ，$C_0(x,\lambda')=\cos\rho x+O(\frac{1}{\rho}\exp(|\zeta|x))$。

(22) ：

$$C_0(x,\lambda')=\cos\rho x+\frac{\sin\rho x}{2\rho}\int_0^x q(t)\mathrm{d}t+\frac{1}{2\rho}\int_0^x q(t)\sin\rho(x-2t)\mathrm{d}t+O(\frac{1}{\rho^2}\exp(|\zeta|x)),\tag{23}$$

$$C'_0(x,\lambda')=-\rho\sin\rho x+\frac{\cos\rho x}{2}\int_0^x q(t)\mathrm{d}t+\frac{1}{2}\int_0^x q(t)\cos\rho(x-2t)\mathrm{d}t+O(\frac{1}{\rho}\exp(|\zeta|x))。\tag{24}$$

，

$$S_0(x,\lambda')=\frac{\sin\rho x}{\rho}-\frac{\cos\rho x}{2\rho^2}\int_0^x q(t)\mathrm{d}t+\frac{1}{2\rho^2}\int_0^x q(t)\cos\rho(x-2t)\mathrm{d}t+O(\frac{1}{\rho^3}\exp(|\zeta|x)),\tag{25}$$

$$S'_0(x,\lambda')=\cos\rho x+\frac{\sin\rho x}{2\rho}\int_0^x q(t)\mathrm{d}t-\frac{1}{2\rho}\int_0^x q(t)\sin\rho(x-2t)\mathrm{d}t+O(\frac{1}{\rho^2}\exp(|\zeta|x)),\tag{26}$$

(21) (23)— (26) ：

$$\begin{cases} A_1 = 1 + (b_2\int_0^{\frac{1}{2}} q(t)\mathrm{d}t - \frac{b}{2})\ \frac{\sin \rho}{\rho} + O(\frac{1}{\rho^2}), \\ B_1 = \frac{b}{2}(1+\cos \rho) + O(\frac{1}{\rho}), \\ A_2 = O(\frac{1}{\rho^2}), \\ B_2 = 1 + O(\frac{1}{\rho}), \end{cases}$$

$\tau(x,\lambda') = C(x,\lambda') + hS(x,\lambda')$， (19)— (21) (23)— (26) ：

$$\tau(x,\lambda') = \cos \rho x + (h + \frac{1}{2}\int_0^x q(t)\mathrm{d}t)\ \frac{\sin \rho x}{\rho} + o(\frac{1}{\rho}\exp(|\ \zeta\ |\ x)), x < \frac{1}{2}, \tag{27}$$

$$\tau(x,\lambda') = \cos \rho x + (h + \frac{1}{2}\int_0^x q(t)\mathrm{d}t + \frac{b}{2})\ \frac{\sin \rho x}{\rho} - \frac{b}{2}\ \frac{\sin \rho(1-x)}{\rho} + o(\frac{1}{\rho}\exp(|\ \zeta\ |\ x)), x > \frac{1}{2}, \tag{28}$$

$$\tau'(x,\lambda') = -\rho\ \sin \rho x + (h + \frac{1}{2}\int_0^x q(t)\mathrm{d}t)\cos \rho x + o(\frac{1}{\rho}\ \exp(|\ \zeta\ |\ x)), x < \frac{1}{2}, \tag{29}$$

$$\tau'(x,\lambda') = -\rho\ \sin \rho x + (h + \frac{1}{2}\int_0^x q(t)\mathrm{d}t + \frac{b}{2})\cos \rho x - \frac{b}{2}\cos \rho(1-x) + o(\frac{1}{\rho}\ \exp(|\ \zeta\ |\ x)), x > \frac{1}{2}, \tag{30}$$

(18)， (28) (30) ：

$$\Delta(\lambda') = \rho\ \sin \rho - \omega_1\ \cos \rho - \omega_2 + o(\exp(|\ \zeta\ |)), \tag{31}$$

：

$$\omega_1 = H + h + \frac{1}{2}\int_0^1 q(t)\mathrm{d}t + \frac{b}{2},\quad \omega_2 = -\frac{b}{2}。 \tag{32}$$

$\lambda_n^0 = (\rho_n^0)^2$

$$\Delta^0(\lambda') = \rho\ \sin \rho \tag{33}$$

，

$$\rho_n = n\pi + o(1), n \to \infty, \tag{34}$$

， (31) ，

$$\sin \rho_n = O(\frac{1}{n})。 \tag{35}$$

(33) ：$\rho_n = n\pi + \varepsilon_n$， $\varepsilon_n \to 0$， $\Delta^0(\lambda_n'^0) = \rho_n^0\ \sin \rho_n^0 = 0$， (35) ：

$$\varepsilon_n\ \cos(n\pi) = O(\frac{1}{n}) + O(\varepsilon_n^2)。$$

(33) ：

$$\Delta_1^0(\lambda_n'^0) := (\frac{\mathrm{d}}{\mathrm{d}x}\Delta^0(\lambda'))_{\lambda'=\lambda_n'^0} = \frac{1}{2\cos(n\pi)} = \frac{1}{2}(-1)^n,$$

$\varepsilon_n\Delta_1^0(\lambda_n'^0) = O(\frac{1}{n}) + O(\varepsilon_n^2)$。

(35) ：

$$\varepsilon_n = O(\frac{1}{n})。 \tag{36}$$

(35) ， (36) ：

$$\rho_n = n\pi + \frac{\theta_n}{n\pi} + \frac{\kappa_n}{n\pi}, \tag{37}$$

:$\kappa_n = o(1)$;$\theta_n = (\omega_1(-1)^n + \omega_2)(2\Delta_1^0(\lambda_n^{'0}))^{-1}$。$\omega_1$ ω_2 (32) , ,

$$\lambda'_n = n^2\pi^2 + \int_0^1 q(x)\mathrm{d}x + H + h + b - (-1)^n b + o(\frac{1}{n})。$$

(15) $e = y'/y$, :

$$\lambda'_n = (n+D)^2\pi^2 + \int_0^1 q(x)\mathrm{d}x - 2[e^*]_0^1 + b - (-1)^n b + o(\frac{1}{n}), \tag{38}$$

D , e ,$e^* = e$, ,$e^* = 0$。

4 , λ_j (38) λ'_j, 。

定理 5 S-L (1)— (5) :

1) $f \in R_N^0$, $n \to \infty$,

$$\lambda_n = \begin{cases} (n-N)^2\pi^2 + \int_0^1 q(x)\mathrm{d}x - b - (-1)^n b + 2\cot\alpha + o(\frac{1}{n}), & \alpha \neq 0, \\ (n+1/2-N)^2\pi^2 + \int_0^1 q(x)\mathrm{d}x - b - (-1)^n b + o(\frac{1}{n}), & \alpha = 0。\end{cases} \tag{39}$$

2) $f \in R_N^+$, $n \to \infty$,

$$\lambda_n = \begin{cases} (n-\frac{1}{2}-N)^2\pi^2 + \int_0^1 q(x)\mathrm{d}x - b - (-1)^n b + 2\cot\alpha + (\frac{2}{a}) + o(\frac{1}{n}), & \alpha \neq 0, \\ (n-N)^2\pi^2 + \int_0^1 q(x)\mathrm{d}x - b - (-1)^n b + (\frac{2}{a}) + o(\frac{1}{n}), & \alpha = 0。\end{cases} \tag{40}$$

3 结 论

[1] , S-L (1)— (5) , Hergloz , S-L S-L 。 , S-L S-L , 。 。 。

参考文献/References:

[1] BENEDEK A I, PANZONE R. On Sturm-Liouville problems with the square root of the eigenvalue parameter conditions contained in the boundary conditions[J]. Notas Algebra Analysis, 1981, 10: 1-62.

[2] BINDING P A, HRYNIV R, LANGER H, et al. Elliptic eigenvalue problems with eigenparameter dependent boundary conditions[J]. J Differential Equations, 2001, 174: 30-54.

[3] DIJKSMA A. Eigenfunction expansions for a class of J-selfadjoint ordinary differential operators with boundary conditions containing the eigenvalue parameter[J]. Proc Roy Soc Edinburgh Sect A, 1980, 86: 1-27.

[4] EBERHARD W, FREILING G, SCHNEIDER A. Eigenfunction expansion for a regular fourth order eigenvalue problem with eigenvalue parameter in the boundary condition[J]. Int J Math Math Sci, 1992, 15: 809-811.

[5] FULTON C T. Singular eigenvalue problems with eigenvalue-parameter contained in the boundary conditions[J]. Proc Roy Soc Edinburgh Sect A, 1980, A 87: 1-34.

[6] HINTON D B, SHAW J K. Differential operators with spectral parameter incompletely in the boundary conditions[J]. Funkcial Ekvac, 1990, 33: 363-385.

[7] RUSSAKOVSKII E M. The matrix Sturm-Liouville problem with spectral parameter in the boundary conditions[J]. Algebraic and operator aspects, Trans Moscow Math Soc, 1996, 57: 159-184.

[8] HINTON D B. Eigenfunction expansions for a singular eigenvalue problem with eigenparameter in the boundary conditions[J]. SIAM J Math Anal, 1981, 12: 572-584.

[9] KOZHEVNIKOV A, YAKUBOV S. On operators generated by elliptic boundary problems with a spectral parameter in boundary conditions[J]. Integral Equations Operator Theory, 1995, 23: 205-231.

[10] RUSSAKOVSKII E M. Operator treatment of boundary problems with spectral parameters entering via ploynomials in the boundary conditions[J]. Funct Anal Appl, 1975, 9: 358-359.

[11] SHKALIKOV A A. Boundary problems for ordinary differential equations with parameter in the boundary conditions[J]. J Soviet Math, 1986, 33: 1311-1342.

[12] TRETTER C. On lambda-nonlinear boundary eigenvalue problems[J]. Mathematics Research Akademie, 1993, 71:1208-1216.

[13] ZAYED E M E, IBRAHIM S F. An expansion theorem for an eigenvalue problem on an arbitrary conditions[J]. Acta Math Sin (Engl Ser), 1995, 11: 399-407.

[14] WEI G, XU H K. Inverse spectral problem with partial information given on the potential and norming constants [J]. Trans Amer Math Soc, 2012, 364: 3265-3288.

[15] PIVOVARCHIK V N. An inverse Sturm-Liouville problem by three spectra[J]. Integral Equ Oper Theory, 1999, 34: 234-243.

[16] GESZTESY F, SIMON B. Inverse spectral analysis with partial information on the potential. II. The case of discrete sprctrum [J]. Trans Amer Math Soc, 2000, 35: 2765-2787.

[17] , , . 2 [J]. ,2006,27(1):15-17.
JIANG Weihua, GUO Yanping, WANG Bin. Existence of two positive solutions to boundary value problems of second order systems[J]. Journal of Hebei University of Science and Technology, 2006,27(1):15-17.

[18] , , . [J]. ,2016,37(6):556-561.
GUO Yanping, MIAO Surong, YU Changlong. Existence of positive solutions to boundary value problem of second-order three-point difference equations on infinite intervals[J]. Journal of Hebei University of Science and Technology, 2016,37(6):556-561.

[19] WEI G, XU H K. Inverse spectral problem for a string equation with partial information [J]. Inverse Problems, 2010, 26: 115004.

[20] GESZTESY F, SIMON B. On the determination of a potential from three spectra[J]. Amer Math Soc Transl, 1997, 2: 18-26.

[21] GESZTESY F, SIMON B. Inverse spectral analysis with partial information on the potential, Ⅰ. The case of an a.c. component in the sprctrum[J]. Helv Phys Acta, 1997, 70: 66-71.

[22] BINDING P A, BROWNE P J, WATSON B A. The Sturm-Liouville problem with the discontinuity conditions at an interior point and boundary conditions depending on the eigenparameter[J]. Proc Edinb Math Soc, 2002, 45: 631-645.

[23] , , . Sturm-Liouville [M]. : , 2015.