

带有输入时滞的 Timoshenko 梁系统的 控制器设计与稳定性分析

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摘要:为了研究输入时滞对 Timoshenko 梁系统稳定性的影响,镇定边界具有输入时滞和载荷的 Timoshenko 梁系统,利用 Backstepping 方法,设计了一种新的控制器来补偿输入时滞带来的影响,从而得到一个稳定的闭环系统。首先,给出一个与原时滞系统等价的无时滞系统;然后,构造一个 Backstepping 线性变换,并证明这个线性变换是有界可逆的;最后,通过这个变换将无时滞系统转化为一个稳定的目标系统,并设计出相应的控制器。结果表明,此无时滞系统与目标系统是等价的,其反馈控制律可以镇定原来的时滞系统。研究方法解决了输入时滞对弹性系统的负面影响,丰富了分布参数控制系统的控制器设计方法及其稳定性理论,在工程实践中具有一定的借鉴意义。

关键词:稳定性理论;Timoshenko 梁;渐近稳定;反馈控制;Backstepping 方法;时滞;载荷

中图分类号:O231.4

MSC(2010)主题分类:34D20

文献标志码:A

Controller design and stability analysis of Timoshenko beam with input delay

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Abstract: In order to study the influence of input time delay on the stability of Timoshenko beam system and stabilize the Timoshenko beam system with input delay and load, by using the Backstepping method, a new controller is designed to compensate for the input delay, and then a stable closed-loop system is obtained. At first, a delay-free system is given, which is equivalent to the original time-delay system. Then, a bounded linear transformation is constructed, and it is proved that the linear transformation is bounded and invertible. Finally, the delay-free system is transformed into a stable target system by the linear transformation, and the corresponding controller is given, which implies that the delay-free system is equivalent to the target system. Therefore, the original time-delay system can be stabilized by the feedback control law. The negative effect of input delay on elastic system is solved by this method, which enriches the controller design method and stability theory of distributed parameter control system, and has a certain theoretical significance in engineering practice.

收稿日期:2017-10-06;修回日期:2017-12-28;责任编辑:张 军

基金项目:国家自然科学基金(61573252)

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韩如梦,刘东毅.带有输入时滞的 Timoshenko 梁系统的控制器设计与稳定性分析[J].河北科技大学学报,2018,39(2):125-134.

HAN Rumeng, LIU Dongyi. Controller design and stability analysis of Timoshenko beam with input delay[J]. Journal of Hebei University of Science and Technology, 2018, 39(2): 125-134.

Keywords: theory of stability; Timoshenko beam; asymptotic stability; feedback control; Backstepping method; time-delay; payload

在航空、海洋和土木等工程领域中,弹性结构通常起着关键性的连接和承载作用。在外部干扰和载荷等的作用下,这些结构会发生振动,给工程结构造成一定的危害。一直以来,很多学者的研究致力于弹性系统的控制器设计与稳定性分析。通过抵制时滞和外部干扰等不利因素来镇定系统^[1-9]。Timoshenko 梁系统考虑了剪切效应和旋转效应的影响,对于弹性杆的动力学行为有着准确的描述,是一种非常精确的系统模型。很多学者对其产生了浓厚的兴趣。本文以一个边界上带有载荷和输入时滞的 Timoshenko 梁为研究对象,利用 Backstepping 方法,设计了一种新的控制器补偿时滞所带来的影响,使闭环系统达到渐近稳定。系统模型如下:

$$\begin{cases} \rho w_{tt}(x,t) - \kappa(w_{xx} - \varphi_x)(x,t) = 0, \\ I_\rho \varphi_{tt}(x,t) - EI\varphi_{xx}(x,t) - \kappa(w_x - \varphi)(x,t) = 0, \\ mw_u(1,t) + \kappa(w_x - \varphi)(1,t) = u_1(t - \tau), \\ J\varphi_u(1,t) + EI\varphi_x(1,t) = u_2(t - \tau), \\ w(0,t) = \varphi(0,t) = 0, \\ w(x,0) = w_0(x), w_t(x,0) = w_1(x), \varphi(x,0) = \varphi_0(x), \varphi_t(x,0) = \varphi_1(x), \\ u_1(\theta) = f_1(\theta), u_2(\theta) = f_2(\theta), \theta \in (-\tau, 0), \end{cases} \quad (1)$$

其中:下标字母表示对应相应变量的偏微分, $x \in (0, 1), t > 0$; 函数 $f_i(\theta)$ 在适当的空间是有界可测的, $i = 1, 2$; $w(x, t)$ 代表梁在其平衡状态下的弹性挠度; $\varphi(x, t)$ 代表总旋转角度; $u_1(t)$ 和 $u_2(t)$ 分别代表边界控制力和力矩; ρ, κ, I_ρ 和 EI 分别代表线密度、剪切弹性模量、梁横截面的惯量矩和刚度系数。

当系统无时滞时,即 $\tau = 0$, 输出反馈控制律:

$$u_1(t) = -\alpha_1 w_t(1, t), \quad u_2(t) = -\alpha_2 \varphi_t(1, t), \quad (2)$$

可以使系统(1)渐近稳定^[10]。

当 $\tau \neq 0$ 时,即系统存在时滞现象时,在哪种反馈控制律的作用下,系统(1)也可以被镇定呢? 这是本文主要考虑的问题。对于时滞系统,文献[11]针对 $au(t) + \beta u(t - \tau)$ 这类控制器研究了一维波方程的稳定性并且得到了所谓的 1/2 法则。文献[12-14]设计了一类新的动态反馈控制器,证明了条件 $|\alpha| \neq |\beta|$ 可以保证所得闭环系统的稳定性。基于 Backstepping 方法^[15-18], 本文设计了一类新的控制器。在该反馈控制律的作用下,所得闭环系统是渐近稳定的。

1 控制器设计与稳定性结论

笔者通过 Backstepping 方法设计原系统的控制器并给出相关的稳定性结论,其主要思想是通过构造一个可逆的有界线性变换,将原系统的稳定性问题转化为目标系统的稳定性问题^[15-18]。

设 $z_i(s, t) = u_i(t + s - \tau), s \in (0, \tau), i = 1, 2$, 那么 $z_i(\tau, t) = u_i(t)$, 并且 $z_i(s, 0) = h_i(s) = u_i(s - \tau)$ 。因此,系统(1)等价于无时滞系统:

$$\begin{cases} \rho w_{tt}(x,t) - \kappa(w_{xx} - \varphi_x)(x,t) = 0, \\ I_\rho \varphi_{tt}(x,t) - EI\varphi_{xx}(x,t) - \kappa(w_x - \varphi)(x,t) = 0, \\ w(0,t) = \varphi(0,t) = 0, \\ mw_u(1,t) + \kappa(w_x - \varphi)(1,t) = z_1(0,t), \\ J\varphi_u(1,t) + EI\varphi_x(1,t) = z_2(0,t), \\ w(x,0) = w_0(x), w_t(x,0) = w_1(x), \\ \varphi(x,0) = \varphi_0(x), \varphi_t(x,0) = \varphi_1(x), \\ z_{it}(s,t) = z_{is}(s,t), \\ z_i(\tau,t) = u_i(t), z_i(s,0) = h_i(s) = u_i(s - \tau) = f_i(s - \tau). \end{cases} \quad (3)$$

根据 Backstepping 方法的思想,首先构建一个线性变换:

$$\begin{cases} \hat{w}(x, t) = w(x, t), \hat{\varphi}(x, t) = \varphi(x, t), \\ v_i(s, t) = z_i(s, t) - \int_0^s q_{i1}(s, r) z_1(r, t) dr - \int_0^s q_{i2}(s, r) z_2(r, t) dr - \int_0^1 \rho \phi_i(s, x) w_t(x, t) dx - \\ \int_0^1 \kappa [\gamma_{ix}(s, x) - \eta_i(s, x)] (w_x(x, t) - \varphi(x, t)) dx - \int_0^1 EI \eta_{ix}(s, x) \varphi_x(x, t) dx - \\ \int_0^1 I_\rho \psi_i(s, x) \varphi_i(x, t) dx - m w_t(1, t) \phi_i(s, 1) - J \varphi_t(1, t) \psi_i(s, 1), \end{cases} \quad (4)$$

下面确定 $q_{i1}(s, r), q_{i2}(s, r), \gamma_i(s, x), \eta_i(s, x), \phi_i(s, x), \psi_i(s, x)$ 。对 $v_i(s, t)$ 分别关于 t 和 s 求偏微分,利用分部积分法得到:

$$\begin{aligned} v_{it}(s, t) = & z_{is}(s, t) - q_{i1}(s, s) z_1(s, t) - q_{i2}(s, s) z_2(s, t) + \int_0^s q_{i1r}(s, r) z_1(r, t) dr + \int_0^s q_{i2r}(s, r) z_2(r, t) dr + \\ & \int_0^1 \kappa [\gamma_{ixx}(s, x) - \eta_{ix}(s, x)] w_t(x, t) dx + \int_0^1 (\kappa [\gamma_{ix}(s, x) - \eta_i(s, x)] + EI \eta_{ixx}(s, x)) \varphi_t(x, t) dx - \\ & \int_0^1 \kappa \phi_i(s, x) [w_{xx}(x, t) - \varphi_{xx}(x, t)] dx - \int_0^1 EI \psi_i(s, x) \varphi_{xx}(x, t) dx + EI \eta_{ix}(s, 1) \varphi_t(1, t) - \\ & \int_0^1 \kappa \psi_i(s, x) [w_x(x, t) - \varphi(x, t)] dx + EI \psi_i(s, 1) \varphi_x(1, t) - \kappa [\gamma_{ix}(s, 1) - \eta_i(s, 1)] w_t(1, t) - \\ & \phi_i(s, 1) \kappa [w_x(1, t) - \varphi(1, t)] + [q_{i1}(s, 0) - \phi_i(s, 1)] z_1(0, t) + [q_{i2}(s, 0) - \psi_i(s, 1)] z_2(0, t) \end{aligned}$$

和

$$\begin{aligned} v_{is}(s, t) = & z_{is}(s, t) - q_{i1}(s, s) z_1(s, t) - q_{i2}(s, s) z_2(s, t) - \int_0^s q_{i1s}(s, r) z_1(r, t) dr - \int_0^s q_{i2s}(s, r) z_2(r, t) dr - \\ & \int_0^1 \rho \phi_{is}(s, x) w_t(x, t) dx - \int_0^1 I_\rho \psi_{is}(s, x) \varphi_t(x, t) dx + \int_0^1 \kappa \gamma_{is}(s, x) [w_{xx}(x, t) - \varphi_{xx}(x, t)] dx + \\ & \int_0^1 EI \eta_{is}(s, x) \varphi_{xx}(x, t) dx + \int_0^1 \kappa \eta_{is}(s, x) [w_x(x, t) - \varphi(x, t)] dx - J \psi_{is}(s, 1) \varphi_t(1, t) - \\ & EI \eta_{is}(s, 1) \varphi_x(1, t) - m \phi_{is}(s, 1) w_t(1, t) - \kappa \gamma_{is}(s, 1) [w_x(1, t) - \varphi(1, t)] + \\ & EI \eta_{is}(s, 0) \varphi_{xx}(0, t) + \kappa \gamma_{is}(s, 0) [w_x(0, t) - \varphi(0, t)]。 \end{aligned}$$

取

$$\begin{cases} q_{i1r}(s, r) + q_{i1s}(s, r) = 0, \quad q_{i2r}(s, r) + q_{i2s}(s, r) = 0, \\ q_{i1}(s, 0) = \phi_i(s, 1) = 0, \quad q_{i2}(s, 0) - \psi_i(s, 1) = 0, \\ \phi_i(s, x) + \gamma_{is}(s, x) = 0, \quad \psi_i(s, x) + \eta_{is}(s, x) = 0, \\ \rho \phi_{is}(s, x) + \kappa [\gamma_{ixx}(s, x) - \eta_{ix}(s, x)] = 0, \\ I_\rho \psi_{is}(s, x) + \kappa [\gamma_{ix}(s, x) - \eta_i(s, x)] + EI \eta_{ixx}(s, x) = 0, \\ EI \eta_{ix}(s, 1) + J \psi_{is}(s, 1) = 0, \quad m \phi_{is}(s, 1) + \kappa [\gamma_{ix}(s, 1) - \eta_i(s, 1)] = 0, \\ \eta_i(s, 0) = 0, \quad \gamma_i(s, 0) = 0, \end{cases}$$

即

$$q_{i1}(s, r) = -\phi_i(s - \gamma, 1), \quad q_{i2}(s, r) = -\psi_i(s - r, 1), \quad \phi_i(s, x) = -\gamma_{is}(s, x), \quad \psi_i(s, x) = -\eta_{is}(s, x), \quad (5)$$

并且 $\gamma_i(s, x)$ 和 $\eta_i(s, x)$ 满足:

$$\begin{cases} \rho \gamma_{is}(s, x) - \kappa [\gamma_{ixx}(s, x) - \eta_{ix}(s, x)] = 0, \\ I_\rho \eta_{is}(s, x) - EI \eta_{ixx}(s, x) - \kappa [\gamma_{ix}(s, x) - \eta_i(s, x)] = 0, \\ \gamma_i(s, 0) = 0, \\ \eta_i(s, 0) = 0, \\ m \gamma_{iss}(s, 1) + \kappa [\gamma_{ix}(s, 1) - \eta_i(s, 1)] = 0, \\ J \eta_{iss}(s, 1) + EI \eta_{ix}(s, 1) = 0, \end{cases} \quad (6)$$

那么 $v_{ii}(s, t) = v_{is}(s, t)$ 。对于变换(4), 令 $s=0$, 得到:

$$\begin{aligned} v_i(0, t) = & z_i(0, t) - \int_0^1 \rho \phi_i(0, x) w_i(x, t) dx - \int_0^1 \kappa(\gamma_{ix}(0, x) - \eta_i(0, x))(w_x(x, t) - \varphi(x, t)) dx - \\ & \int_0^1 EI \eta_{ix}(0, x) \varphi_x(x, t) dx - \int_0^1 I_\rho \psi_i(0, x) \varphi_t(x, t) dx - \\ & m w_i(1, t) \phi_i(0, 1) - J \varphi_t(1, t) \psi_i(0, 1)。 \end{aligned} \quad (7)$$

取 $\gamma_i(s, x)$ 和 $\eta_i(s, x)$ 的初值为

$$\gamma_1(0, x) = 0, \eta_1(0, x) = 0, \gamma_{1s}(0, x) = 0, \eta_{1s}(0, x) = 0, \gamma_{1s}(0, 1) = -m^{-1} \alpha_1, \eta_{1s}(0, 1) = 0, \quad (8)$$

$$\gamma_2(0, x) = 0, \eta_2(0, x) = 0, \gamma_{2s}(0, x) = 0, \eta_{2s}(0, x) = 0, \gamma_{2s}(0, 1) = 0, \eta_{2s}(0, 1) = -J^{-1} \alpha_2, \quad (9)$$

从而由式(7)–式(9)可知,

$$\begin{cases} z_1(0, t) = -\alpha_1 w_i(1, t) + v_1(0, t), \\ z_2(0, t) = -\alpha_2 w_i(1, t) + v_2(0, t), \end{cases} \quad (10)$$

于是, 变换(4)将系统(3)转化为如下目标系统:

$$\begin{cases} \rho \hat{w}_{tt}(x, t) - \kappa(\hat{w}_{xx} - \hat{\varphi}_x)(x, t) = 0, \\ I_\rho \hat{\varphi}_{tt}(x, t) - EI \hat{\varphi}_{xx}(x, t) - \kappa(\hat{w}_x - \hat{\varphi})(x, t) = 0, \\ \hat{w}(0, t) = \hat{\varphi}(0, t) = 0, \\ m \hat{w}_{it}(s, 1) + \kappa(\hat{w}_x - \hat{\varphi})(1, t) = -\alpha_1 \hat{w}_i(1, t) + v_1(0, t), \\ J \hat{\varphi}_{it}(s, 1) + EI \hat{\varphi}_x(1, t) = -\alpha_2 \hat{\varphi}_i(1, t) + v_2(0, t), \\ \hat{w}(x, 0) = \hat{w}_0(x) = w_0(x), \hat{w}_t(x, 0) = \hat{w}_1(x) = w_1(x), \\ \hat{\varphi}(x, 0) = \hat{\varphi}_0(x) = \varphi_0(x), \\ \hat{\varphi}_t(x, 0) = \hat{\varphi}_1(x) = \varphi_1(x), \\ v_{ii}(s, t) = v_{is}(s, t), \\ v_i(\tau, t) = 0, v_i(s, 0) = v_{i0}(s), \end{cases} \quad (11)$$

其中 $x \in (0, 1), t > 0, s \in (0, \tau)$ 并且

$$\begin{aligned} v_{i0}(s) = & h_i(s) - \int_0^s q_{i1}(s, r) h_1(r) dr - \int_0^s q_{i2}(s, r) h_2(r) dr - \\ & \int_0^1 \rho \phi_i(s, x) w_1(x) dx - \int_0^1 \kappa(\gamma_{ix}(s, x) - \eta_i(s, x))(w'_0(x) - \varphi_0(x)) dx - \\ & \int_0^1 EI \eta_{ix}(s, x) \varphi'_0(x) dx - \int_0^1 I_\rho \psi_i(s, x) \varphi_1(x) dx - \\ & m w_1(1) \phi_i(s, 1) - J \varphi_1(1) \psi_i(s, 1)。 \end{aligned} \quad (12)$$

对于目标系统有如下定理。

定理 1 假设 $h \in L^2(0, \tau), (w_0, w_1, \varphi_0, \varphi_1) \in \mathcal{H}$, 其中 \mathcal{H} 是系统的状态空间(见 2.1 节), 那么目标系统(11)–(12)是渐近稳定的。

对于变换(4)有以下结论。

定理 2 设 $\gamma_i(s, x)$ 和 $\eta_i(s, x)$ 是式(6), 式(8)和式(9)给出, 函数 $q_{i1}(s, r), q_{i2}(s, r), \phi_i(s, x), \psi_i(s, x)$ 由式(5)确定, 那么线性变换(4)是有界可逆的。

令 $t = \tau$, 由式(4)可知, 系统的控制律为

$$\begin{aligned} u_i(t) = z_i(\tau, t) = & - \int_0^\tau \gamma_{is}(r, 1) u_1(t-r) dr - \\ & \int_0^\tau \eta_{is}(r, 1) u_2(t-r) dr - \int_0^1 \rho \gamma_{is}(\tau, x) w_i(x, t) dx + \\ & \int_0^1 \kappa(\gamma_{ix}(\tau, x) - \eta_i(\tau, x))(w_x(x, t) - \varphi(x, t)) dx + \\ & \int_0^1 EI \eta_{ix}(\tau, x) \varphi_x(x, t) dx - \int_0^1 I_\rho \eta_{is}(\tau, x) \varphi_t(x, t) dx - \\ & m w_i(1, t) \gamma_{is}(s, 1) - J \varphi_t(1, t) \eta_{is}(s, 1), \quad i = 1, 2, \end{aligned} \quad (13)$$

其中 $\gamma_i(s, x)$ 和 $\eta_i(s, x)$ 由式(6)、式(8)和式(9)确定。

由上述可知,当线性变换(4)存在有界的逆变换时,无时滞系统(3)与目标系统(11)–(12)等价。

定理 3 假设 $h \in L^2(0, \tau)$, $(w_0, w_1, \varphi_0, \varphi_1) \in \mathcal{H}$, 那么在反馈控制律(13)的作用下,原系统(1)是渐近稳定的。

2 主要结论的证明

2.1 预备知识

首先,引进空间 $\mathcal{V} = L^2(0, \tau)$ 和 $H_E^k(0, 1) = \{f \in H^k(0, 1) \mid f(0) = 0\}$, $k = 1, 2$ 。记 $\mathcal{R} \times \mathcal{R}$ 为实数空间。取原系统(1)的状态空间为 $\mathcal{H} = H_E^1(0, 1) \times L^2(0, 1) \times H_E^1(0, 1) \times L^2(0, 1) \times \mathcal{R} \times \mathcal{R}$, 其内积为

$$\langle (w, z, \varphi, \psi, \zeta, \theta)^T, (f, g, h, q, \xi, \nu)^T \rangle_{\mathcal{H}} = \int_0^1 [\kappa(w'(x) - \varphi(x))(f'(x) - h(x)) + \rho z(x)g(x) + EI\varphi'(x)h'(x) + I_\rho \psi(x)q(x)] dx + m\zeta\xi + J\theta\nu,$$

显然, \mathcal{H} 是一个 Hilbert 空间。

对于系统(1),在空间 \mathcal{H} 上定义一个算子 A_0 :

$$A_0(w, z, \varphi, \psi, \zeta, \theta)^T = \left(z, \frac{\kappa(w_{xx} - \varphi_x)}{\rho}, \psi, \frac{EI\varphi_{xx}}{I_\rho} + \frac{\kappa(w_x - \varphi)}{I_\rho}, \frac{-\kappa(w_x - \varphi)(1)}{m}, \frac{-EI\varphi_x(1)}{J} \right)^T, \tag{14}$$

定义域为

$$D(A_0) = \{(w, z, \varphi, \psi, \zeta, \theta)^T \in H_E^2(0, 1) \times H_E^1(0, 1) \times H_E^2(0, 1) \times H_E^1(0, 1) \times \mathcal{R} \times \mathcal{R} \mid \zeta = z(1), \theta = \psi(1)\}.$$

另外,定义一个算子 $B: \mathcal{U} \rightarrow \mathcal{H}$,

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 1/m & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/J \end{pmatrix}^T, \quad \mathcal{U} = \mathcal{R} \times \mathcal{R}. \tag{15}$$

那么,系统(1)可以写成如下抽象发展方程的形式:

$$\frac{dX(\cdot, t)}{dt} = A_0 X(\cdot, t) + BU(t - \tau), \quad X(\cdot, 0) = X_0(\cdot), \quad t > 0, \tag{16}$$

其中:

$$X(\cdot, t) = (w(\cdot, t), w_t(\cdot, t), \varphi(\cdot, t), \varphi_t(\cdot, t), w_t(1, t), \varphi_t(1, t))^T, \\ U(t - \tau) = (u_1(t - \tau), u_2(t - \tau))^T.$$

类似地,式(6)和式(8)(或式(9))也可以写成如下抽象发展方程的形式:

$$\begin{cases} \frac{dW_i(\cdot, s)}{ds} = A_0 W_i(\cdot, s), & 0 < s < \tau, \\ W_1(\cdot, 0) = (0, \gamma_{1s}(\cdot, 0), 0, 0, -m^{-1}\alpha_1, 0)^T \text{ (或 } W_2(0, \cdot) = (0, 0, 0, \eta_{2s}(\cdot, 0), 0, -J^{-1}\alpha_2)^T), \end{cases} \tag{17}$$

其中 $W_i(\cdot, s) = (\gamma_i(\cdot, s), \gamma_{is}(\cdot, s), \eta_i(\cdot, s), \eta_{is}(\cdot, s), \gamma_{is}(1, s), \eta_{is}(1, s))^T$ 。

对于算子 A_0 , 经过简单的计算,利用 Stone 定理^[19],可以得到如下引理。

引理 1 由式(14)定义的算子 A_0 是斜自伴的,并且在 \mathcal{H} 中生成酉群 $e^{A_0 t}$ 。

目标系统(11)可分为 2 个部分:传输系统

$$\begin{cases} v_{it}(s, t) = v_{is}(s, t), \\ v_i(\tau, t) = 0, \\ v_i(s, 0) = v_{i0}(s) \end{cases} \tag{18}$$

和 Timoshenko 梁系统

$$\begin{cases} \rho \hat{w}_{tt}(x,t) - \kappa(\hat{w}_{xx} - \hat{\varphi}_x)(x,t) = 0, \\ I_\rho \hat{\varphi}_{tt}(x,t) - EI \hat{\varphi}_{xx}(x,t) - \kappa(\hat{w}_x - \hat{\varphi})(x,t) = 0, \\ \hat{w}(0,t) = \hat{\varphi}(0,t) = 0, \\ m \hat{w}_t(1,t) + \kappa(\hat{w}_x - \hat{\varphi})(1,t) = -\alpha_1 \hat{w}_t(1,t) + v_1(0,t), \\ J \hat{\varphi}_t(1,t) + EI \hat{\varphi}_x(1,t) = -\alpha_2 \hat{\varphi}_t(1,t) + v_2(0,t), \\ \hat{w}(x,0) = \hat{w}_0(x), \hat{w}_t(x,0) = \hat{w}_1(x), \hat{\varphi}(x,0) = \hat{\varphi}_0(x), \hat{\varphi}_t(x,0) = \hat{\varphi}_1(x). \end{cases} \quad (19)$$

在空间 \mathcal{H} 上定义一个算子 \mathbf{A} :

$$\mathbf{A}(w, z, \varphi, \psi, \zeta, \theta)^\top = \left(z, \frac{\kappa(w_{xx} - \varphi_x)}{\rho}, \psi, \frac{EI\varphi_{xx} + \kappa(w_x - \varphi)}{I_\rho}, -\frac{1}{m}[\kappa(w_x - \varphi)(1) + \alpha_1 \zeta], -\frac{1}{J}[EI\varphi_x(1) + \alpha_2 \theta] \right)^\top. \quad (20)$$

定义域为

$$\mathbf{D}(\mathbf{A}) = \{(w, z, \varphi, \psi, \zeta, \theta)^\top \in H_E^2(0,1) \times H_E^1(0,1) \times H_E^2(0,1) \times H_E^1(0,1) \times \mathcal{R} \times \mathcal{R} \mid \zeta = z(1), \theta = \psi(1)\},$$

此时,系统(18)和系统(19)的抽象发展方程分别为

$$\frac{dv_i(\cdot, t)}{dt} = \mathcal{F}v_i(\cdot, t), \quad v_i(\cdot, 0) = v_{i0}(\cdot), \quad t > 0 \quad (21)$$

和

$$\frac{d\hat{\mathbf{X}}(\cdot, t)}{dt} = \mathbf{A}\hat{\mathbf{X}}(\cdot, t) + \mathbf{B}\mathbf{V}(0, t), \quad \hat{\mathbf{X}}(\cdot, 0) = \mathbf{X}_0(\cdot), \quad t > 0. \quad (22)$$

其中,算子 $\mathcal{F}v = \frac{dv}{ds}$, $\mathbf{D}(\mathbf{F}) = H^1(0, \tau)$, $\hat{\mathbf{X}}(\cdot, t) = (\hat{w}(\cdot, t), \hat{w}_t(\cdot, t), \hat{\varphi}(\cdot, t), \hat{\varphi}_t(\cdot, t), \hat{w}_t(1, t), \hat{\varphi}_t(1, t))^\top$, $\mathbf{V}(s, t) = (v_1(s, t), v_2(s, t))^\top$.

经过计算可得:

$$\begin{aligned} \langle \mathbf{A}(w, z, \varphi, \psi, \zeta, \theta)^\top, (w, z, \varphi, \psi, \zeta, \theta)^\top \rangle_{\mathcal{H}} = & \int_0^1 \{ \kappa(z_x(x) - \psi(x))(w_x(x) - \varphi(x)) + \kappa(w_{xx}(x) - \varphi_x(x))z(x) + EI\psi_x(x)\varphi_x(x) + \\ & [EI\varphi_{xx}(x) + \kappa(w_x(x) - \varphi(x))]\psi(x) \} dx - [\kappa(w_x - \varphi)(1) + \alpha_1 \zeta]\zeta - [EI\varphi_x(1) + \alpha_2 \theta]\theta = \\ & -\alpha_1 \zeta^2 - \alpha_2 \theta^2 \leq 0, \end{aligned}$$

故, \mathbf{A} 是耗散的。显然, \mathbf{A} 是闭稠定线性算子且 $0 \in \rho(\mathbf{A})$ 。由文献[10]可以得到如下引理。

引理 2 由式(20)定义的算子 \mathbf{A} 在空间 \mathcal{H} 上生成一个渐近稳定 C_0 半群, 记为 e^{At} 。

因为

$$\begin{aligned} \langle \mathbf{A}(w, z, \varphi, \psi, \zeta, \theta)^\top, (f, g, h, q, \xi, \nu)^\top \rangle_{\mathcal{H}} = & \int_0^1 \{ \kappa(z_x(x) - \psi(x))(f_x(x) - h(x)) + \kappa(w_{xx}(x) - \varphi_x(x))g(x) + \\ & EI\psi_x(x)h_x(x) + [EI\varphi_{xx}(x) + \kappa(w_x(x) - \varphi(x))]q(x) \} dx - \\ & [\kappa(w_x - \varphi)(1) + \alpha_1 \zeta]\xi - [EI\varphi_x(1) + \alpha_2 \theta]\nu = \\ & - \int_0^1 \{ \kappa(w_x(x) - \varphi(x))(g_x(x) - q(x)) + \kappa z(x)(f_{xx}(x) - h_x(x)) + \\ & EI\varphi_x(x)q_x(x) + \psi(x)[EIh_{xx}(x) + k(f_x(x) - h(x))]\} dx + \\ & [\kappa(f_x - h)(1) - \alpha_1 \xi]\zeta + [EIh_x(1) - \alpha_2 \nu]\theta = \\ & \langle (w, z, \varphi, \psi, \zeta, \theta)^\top, \mathbf{A}^*(f, g, h, q, \xi, \nu)^\top \rangle_{\mathcal{H}}, \end{aligned} \quad (23)$$

所以, 伴随算子 \mathbf{A}^* 的定义如下:

$$\mathbf{A}^*(w, z, \varphi, \psi, \zeta, \theta)^\top = \left(-z, -\frac{\kappa(w_{xx} - \varphi_x)}{\rho}, -\psi, -\frac{EI\varphi_{xx} + \kappa(w_x - \varphi)}{I_\rho}, \frac{\kappa(w_x - \varphi)(1) - \alpha_1 \zeta}{m}, \frac{EI\varphi_x(1) - \alpha_2 \theta}{J} \right)^\top, \quad (24)$$

定义域为

$$D(\mathbf{A}^*) = \{(\omega, z, \varphi, \psi, \zeta, \theta)^\top \in H_E^2(0, 1) \times H_E^1(0, 1) \times H_E^2(0, 1) \times H_E^1(0, 1) \times \mathcal{R} \times \mathcal{R} \mid \zeta = z(1), \theta = \psi(1)\}.$$

笔者给出一个引理。

引理 3 算子 \mathbf{B}^* 是 e^{A^*t} 的一个可容许观测算子, 即 B 是 e^{At} 的一个可容许控制算子。其中算子 $\mathbf{A}, \mathbf{A}^*, \mathbf{B}$ 分别由式(20), 式(24) 和式(15) 定义, $\mathbf{B}^* = \mathbf{B}^\top$ 。

证明 根据式(24) 可知, 系统(22) 的伴随系统为

$$\begin{cases} \frac{d\hat{\mathbf{X}}^*(\cdot, t)}{dt} = \mathbf{A}^* \hat{\mathbf{X}}^*(\cdot, t), & t > 0, \\ \hat{\mathbf{X}}^*(\cdot, 0) = \hat{\mathbf{X}}_0^*, \quad \hat{\mathbf{Y}}^*(t) = \mathbf{B}^* \hat{\mathbf{X}}^*(\cdot, t), \end{cases}$$

即

$$\begin{cases} \rho \hat{w}_t^*(x, t) - \kappa(\hat{w}_{xx}^* - \hat{\varphi}_x^*)(x, t) = 0, & 0 < x < 1, t > 0, \\ I_\rho \hat{\varphi}_t^*(x, t) - EI \hat{\varphi}_{xx}^*(x, t) - \kappa(\hat{w}_x^* - \hat{\varphi}^*)(x, t) = 0, \\ \hat{w}^*(0, t) = \hat{\varphi}^*(0, t) = 0, \\ m \hat{w}_t^*(1, t) + \kappa(\hat{w}_x^* - \hat{\varphi}^*)(1, t) = -\alpha_1 \hat{w}_t^*(1, t), \\ J \hat{\varphi}_t^*(1, t) + EI \hat{\varphi}_x^*(1, t) = -\alpha_2 \hat{\varphi}_t^*(1, t), \\ \hat{w}^*(x, 0) = \hat{w}_0^*(x), \\ \hat{w}_t^*(x, 0) = \hat{w}_1^*(x), \\ \hat{\varphi}^*(x, 0) = \hat{\varphi}_0^*(x), \\ \hat{\varphi}_t^*(x, 0) = \hat{\varphi}_1^*(x), \\ \hat{y}^*(t) = (\hat{w}_t(1)/m, \hat{\varphi}_t(1)/J)^\top. \end{cases}$$

利用分部积分做简单的计算得到:

$$\begin{aligned} \frac{d}{dt} \|\hat{\mathbf{X}}^*(t)\|_{\mathcal{X}}^2 &= \frac{d}{dt} \|(\hat{w}^*, \hat{w}_t^*, \hat{\varphi}^*, \hat{\varphi}_t^*, \hat{w}_t^*(1), \hat{\varphi}_t^*(1))^\top\|_{\mathcal{X}}^2 = \\ &2 \int_0^1 [\kappa(\hat{w}_x^*(x) - \hat{\varphi}^*(x))(\hat{w}_{xt}^*(x) - \hat{\varphi}_t^*(x)) + \rho \hat{w}_t^*(x) \hat{w}_t^*(x) + EI \hat{\varphi}_x^*(x) \hat{\varphi}_{xt}^*(x) + I_\rho \hat{\varphi}_t^*(x) \hat{\varphi}_t^*(x)] dx + \\ &2m \hat{w}_t^*(1) \hat{w}_t^*(1) + 2J \hat{\varphi}_t^*(1) \hat{\varphi}_t^*(1) = \\ &2 \int_0^1 [\kappa(\hat{w}_x^*(x) - \hat{\varphi}^*(x))(\hat{w}_{xt}^*(x) - \hat{\varphi}_t^*(x)) + \hat{w}_t^*(x) \kappa(\hat{w}_{xx}^* - \hat{\varphi}_x^*)(x, t) + \\ &EI \hat{\varphi}_x^*(x) \hat{\varphi}_{xt}^*(x) + \hat{\varphi}_t^*(x) (EI \hat{\varphi}_{xx}^*(x, t) + \kappa(\hat{w}_x^* - \hat{\varphi}^*)(x, t))] dx - \\ &2 \hat{w}_t^*(1, t) [\kappa(\hat{w}_x^* - \hat{\varphi}^*)(1, t) + \alpha_1 \hat{w}_t^*(1, t)] - 2 \hat{\varphi}_t^*(1, t) [EI \hat{\varphi}_x^*(1, t) + \alpha_2 \hat{\varphi}_t^*(1, t)] = \\ &-2\alpha_1 \hat{w}_t^{*2}(1, t) - 2\alpha_2 \hat{\varphi}_t^{*2}(1, t) \leq 0, \end{aligned}$$

那么

$$\begin{aligned} \int_0^t |\mathbf{B}^* e^{A^*t} \hat{\mathbf{X}}_0^*|^2 dt &= \int_0^t |\mathbf{B}^* \hat{\mathbf{X}}^*(t)|^2 dt = \\ &\int_0^t \left| \frac{\hat{w}_t^{*2}(1, t)}{m^2} + \frac{\hat{\varphi}_t^{*2}(1, t)}{J^2} \right|^2 dt \leq \\ &\frac{1}{2 \min \{m^2 \alpha_1, J^2 \alpha_2\}} \int_0^t \frac{d}{dt} \|\hat{\mathbf{X}}^*(t)\|_{\mathcal{X}}^2 dt \leq \\ &\frac{1}{2 \min \{m^2 \alpha_1, J^2 \alpha_2\}} \|\hat{\mathbf{X}}_0^*\|_{\mathcal{X}}^2, \end{aligned}$$

这表明算子 \mathbf{B}^* 是 e^{A^*t} 的一个可容许观测算子, 即 \mathbf{B} 是 e^{At} 的一个可容许控制算子。证毕。

2.2 主要结论的证明

定理 1 的证明。

证明 由 2.1 节内容可知, 系统(11)和系统(12)被分解为系统(18)和系统(19)。当 $t \geq \tau$ 时, $V(s, t) = 0$ 。故只

需考虑系统(19)的稳定性。由引理 1 知,系统(17)在 \mathcal{H} 上是适定的,并且 $\|W_i(s)\|_{\mathcal{H}} = \|W_i(0)\|_{\mathcal{H}}, \forall 0 \leq s \leq \tau$ 。

记 $M = (M_1 + 1)M_2$, 其中 $M_1 = \max\{1/m, 1/J, 1\}\tau$ 和 $M_2 = \|W_1(0)\|_{\mathcal{H}} + \|W_2(0)\|_{\mathcal{H}}$ 。由式(5)可知, $q_{i1}(s, r) = -\phi_i(s - r, 1), q_{i2}(s, r) = -\psi_i(s - r, 1)$, 所以:

$$\int_0^s q_{i1}(s, r)h_1(r, t)dr + \int_0^s q_{i2}(s, r)h_2(r, t)dr \leq M_1 \|W_i(0)\|_{\mathcal{H}} \left(\int_0^\tau |h_1(s)|^2 ds + \int_0^\tau |h_2(s)|^2 ds \right)。$$

对于式(12)进行适当的放缩,得到:

$$\begin{aligned} \int_0^\tau |v_{i0}(s)|^2 ds &\leq M_3 \left[\int_0^\tau |h_i(s)|^2 ds + \int_0^\tau \left| \int_0^s q_{i1}(s, r)h_1(r)dr \right|^2 ds + \right. \\ &\quad \left. \int_0^\tau \left| \int_0^s q_{i2}(s, r)h_2(r)dr \right|^2 ds + \int_0^\tau \left| \int_0^1 \rho\phi_i(s, x)\omega_1(x)dx \right|^2 ds + \right. \\ &\quad \left. \int_0^\tau \left| \int_0^1 \kappa(\gamma_{ix}(s, x) - \eta_i(s, x))(\omega'_0(x) - \varphi_0(x))dx \right|^2 ds + \right. \\ &\quad \left. \int_0^\tau \left| \int_0^1 E \eta_{ix}(s, x)\varphi'_0(x)dx \right|^2 ds + \int_0^\tau \left| \int_0^1 \psi_i(s, x)\varphi_1(x)dx \right|^2 ds + \right. \\ &\quad \left. \int_0^\tau |m\omega_1(1)\phi_i(s, 1)|^2 ds + \int_0^\tau |J\varphi_1(1)\psi_i(s, 1)|^2 ds \right] \leq \\ &M_3 \left[\int_0^\tau |h_i(s)|^2 ds + \tau M \left(\int_0^\tau |h_1(r)|^2 dr + \int_0^\tau |h_2(r)|^2 dr + \|X_0\|^2 \right) \right], \end{aligned}$$

其中 M_3 为正常数。这表明当 $h_i \in L^2(0, \tau)$ 和 $X_0 \in \mathcal{H}$ 时,有 $v_{i0} \in L^2(0, \tau)$ 。显然, \mathcal{F} 在空间 \mathcal{V} 上生成 C_0 半群 $e^{\mathcal{F}t}$ (见文献[20],例 2.3.8)。相应地,系统(21)的温和解^[19,21]为

$$v_i(s, t) = e^{\mathcal{F}t}v_{i0}(s) = \begin{cases} 0, & s + t \geq \tau, \\ v_{i0}(s + t), & s + t \in [0, \tau). \end{cases}$$

由引理 3 知, \mathbf{B} 是 e^{At} 的一个可容许控制算子。因此,系统(19)在 \mathcal{H} 上是适定的,其解可以表示为

$$\hat{X}(x, t) = e^{At}\hat{X}_0(x) + \int_0^t e^{A(t-r)}\mathbf{B}\mathbf{V}(0, r)dr。$$

当 $t \geq \tau$ 时, $V(s, t) = 0$, 所以

$$\hat{X}(x, t) = e^{A(t-\tau)}e^{At}\hat{X}_0(x) + \int_0^d e^{A(t-\tau+r-r)}\mathbf{B}\mathbf{V}(0, r)dr = e^{A(t-\tau)}\hat{X}(x, \tau)。$$

这表明目标系统是渐近稳定的。证毕。

证明定理 2。

证明 由式(4)可知,只需考虑变换:

$$\begin{aligned} v_i(s, t) &= z_i(s, t) - \int_0^s q_{i1}(s, r)z_1(r, t)dr - \int_0^s q_{i2}(s, r)z_2(r, t)dr - \\ &\quad \int_0^1 \rho\phi_i(s, x)\omega_t(x, t)dx - \int_0^1 \kappa(\gamma_{ix}(s, x) - \eta_i(s, x))(\omega_x(x, t) - \varphi(x, t))dx - \\ &\quad \int_0^1 EI\eta_{ix}(s, x)\varphi_x(x, t)dx - \int_0^1 I_\rho\psi_i(s, x)\varphi_t(x, t)dx - \\ &\quad m\omega_t(1, t)\phi_i(s, 1) - J\varphi_t(1, t)\psi_i(s, 1) \end{aligned} \tag{25}$$

即可。

$$\text{记 } Z(s, t) = \begin{pmatrix} z_1(s, t) \\ z_2(s, t) \end{pmatrix}, \quad G(s, t) = \begin{pmatrix} g_1(s, t) \\ g_2(s, t) \end{pmatrix},$$

其中:

$$\begin{aligned} g_i(s, t) &= - \int_0^s q_{i1}(s, r)z_1(r, t)dr - \int_0^s q_{i2}(s, r)z_2(r, t)dr - \\ &\quad \int_0^1 \rho\phi_i(s, x)\omega_t(x, t)dx - \int_0^1 \kappa(\gamma_{ix}(s, x) - \eta_i(s, x))(\omega_x(x, t) - \varphi(x, t))dx - \\ &\quad \int_0^1 EI\eta_{ix}(s, x)\varphi_x(x, t)dx - \int_0^1 I_\rho\psi_i(s, x)\varphi_t(x, t)dx - \\ &\quad m\omega_t(1, t)\phi_i(s, 1) - J\varphi_t(1, t)\psi_i(s, 1), \end{aligned}$$

那么式(25)可以表示为 $V(s, t) = Z(s, t) + G(s, t)$, 即 $v_i(s, t) = z_i(s, t) + g_i(s, t)$, 从而 $Z(s, t) = V(s, t) - G(s, t)$, 即 $z_i(s, t) = v_i(s, t) - g_i(s, t)$ 。

因此

$$g_i(s, t) = - \int_0^s q_{i1}(s, r)[v_1(r, t) - g_1(r, t)]dr - \int_0^s q_{i2}(s, r)[v_2(r, t) - g_2(r, t)]dr - \int_0^1 \rho \phi_i(s, x) \omega_t(x, t) dx - \int_0^1 \kappa(\gamma_{ix}(s, x) - \eta_i(s, x))(\omega_x(x, t) - \varphi(x, t)) dx - \int_0^1 EI \eta_{ix}(s, x) \varphi_x(x, t) dx - \int_0^1 I_\rho \psi_i(s, x) \varphi_t(x, t) dx - m\omega_t(1, t) \phi_i(s, 1) - J\varphi_t(1, t) \psi_i(s, 1)。$$

为了证明该方程存在唯一解, 设 $G_0(s, t) = \begin{pmatrix} g_{1,0}(s, t) \\ g_{2,0}(s, t) \end{pmatrix}$ 和 $G_n(s, t) = \begin{pmatrix} g_{1,n}(s, t) \\ g_{2,n}(s, t) \end{pmatrix}$,

其中:

$$g_{i,n}(s, t) = \int_0^s q_{i1}(s, r)g_{1,n-1}(r, t)dr + \int_0^s q_{i2}(s, r)g_{2,n-1}(r, t)dr, \quad n = 1, 2, \dots$$

和

$$g_{i,0}(s, t) = - \int_0^s q_{i1}(s, r)v_1(r, t)dr - \int_0^s q_{i2}(s, r)v_2(r, t)dr - \int_0^1 \rho \phi_i(s, x) \omega_t(x, t) dx - \int_0^1 \kappa(\gamma_{ix}(s, x) - \eta_i(s, x))(\omega_x(x, t) - \varphi(x, t)) dx - \int_0^1 EI \eta_{ix}(s, x) \varphi_x(x, t) dx - \int_0^1 I_\rho \psi_i(s, x) \varphi_t(x, t) dx - m\omega_t(1, t) \phi_i(s, 1) - J\varphi_t(1, t) \psi_i(s, 1),$$

那么,

$$\begin{aligned} |g_{i,0}(s, t)| &\leq \left| \int_0^s q_{i1}(s, r)v_1(r, t)dr \right| + \left| \int_0^s q_{i2}(s, r)v_2(r, t)dr \right| + \\ &\quad \left| \int_0^1 \rho \phi_i(s, x) \omega_t(x, t) dx \right| + \left| \int_0^1 \kappa(\gamma_{ix}(s, x) - \eta_i(s, x))(\omega_x(x, t) - \varphi(x, t)) dx \right| + \\ &\quad \left| \int_0^1 EI \eta_{ix}(s, x) \varphi_x(x, t) dx \right| + \left| \int_0^1 I_\rho \psi_i(s, x) \varphi_t(x, t) dx \right| + \\ &\quad \left| m\omega_t(1, t) \phi_i(s, 1) \right| + \left| J\varphi_t(1, t) \psi_i(s, 1) \right| \leq \\ &\quad M(\|X\|_{\mathcal{X}} + \|V\|_{\mathcal{Y} \times \mathcal{Y}})。 \end{aligned}$$

类似地, 可以得到:

$$\begin{cases} |g_{i,1}(s, t)| \leq M^2(\|X\|_{\mathcal{X}} + \|V\|_{\mathcal{Y} \times \mathcal{Y}})s, \\ |g_{i,2}(s, t)| \leq M^3(\|X\|_{\mathcal{X}} + \|V\|_{\mathcal{Y} \times \mathcal{Y}}) \frac{s^2}{2!}, \\ \vdots \\ |g_{i,n}(s, t)| \leq M^{n+1}(\|X\|_{\mathcal{X}} + \|V\|_{\mathcal{Y} \times \mathcal{Y}}) \frac{s^n}{n!}。 \end{cases}$$

这些估计表明级数 $G(s, \bullet) = \sum_{n=0}^{\infty} G_n(s, \bullet) = \begin{pmatrix} \sum_{n=0}^{\infty} g_{1,n}(s, \bullet) \\ \sum_{n=0}^{\infty} g_{2,n}(s, \bullet) \end{pmatrix}$ 关于 $s \in [0, \tau]$ 绝对一致收敛并且级数和

为系统(4)的一个解。此外, 存在一个常数 $C > 0$ 使得 $\|G(s, \bullet)\| \leq C(\|X\|_{\mathcal{X}} + \|V\|_{\mathcal{Y} \times \mathcal{Y}})$ 。这意味着存

在一个有界线性算子 $\Phi: \mathcal{X} \times \mathcal{Y} \times \mathcal{Y} \rightarrow \mathcal{Y} \times \mathcal{Y}$, 使得 $G(s, \bullet) = \left(\Phi \begin{pmatrix} X \\ V \end{pmatrix} \right) (s, \bullet)$ 。那么有:

$$\begin{cases} \hat{X}(s, t) = X(s, t), \\ \hat{Z}(s, t) = V(s, t) - \left(\Phi \begin{pmatrix} X \\ V \end{pmatrix} \right) (s, t), \end{cases}$$

所以变换(4)是有界可逆的。证毕。

最后, 证明定理 3。

证明 由定理 2 易知, 无时滞系统(3)与目标系统(11)和系统(12)是等价的。由引理 2 知, 目标系统是渐近稳定的, 故无时滞系统(3)也是渐近稳定的, 即反馈控制律(13)可以使得原系统(1)渐近稳定。证毕。

3 结 论

基于 Backstepping 方法, 针对边界带有载荷和输入时滞的 Timoshenko 梁系统设计了一个新的控制器, 证明了原系统在这个反馈控制律作用下是渐近稳定的。研究重点在于控制器的设计与稳定性分析, 难点在于目标系统的构造和线性变换的选取。本文考虑的控制算子是有界的, 当控制算子是无界的时候, 应该如何考虑? 这类控制器是否可以应用到高维系统模型中? 这都是将来要研究的问题。

参考文献/References:

- [1] MORGÜL Ö, RAO B P, CONRAD F. On the stabilization of a cable with a tip mass[J]. IEEE Transactions on Automatic Control, 2002, 39(10): 2140-2145.
- [2] BAICU C F, RAHN C D, DAWSON D M. Exponentially stabilizing boundary control of string-mass systems[J]. Journal of Vibration and Control, 1999, 5(3): 491-502.
- [3] D'ANDRÉA-NOVEL B, BOUSTANY F, CONRAD F. Control of an overhead crane: Stabilization of flexibilities[J]. Boundary Control and Boundary Variation, 1992, 178(1): 1-26.
- [4] D'ANDRÉA-NOVEL B, BOUSTANY F, CONRAD F, et al. Feedback stabilization of a hybrid PDE-ODE system: Application to an overhead crane[J]. Mathematics of Control, Signals, and Systems, 1994, 7(1): 1-22.
- [5] GUO Baozhu, XU Chengzhong. On the spectrum-determined growth condition of a vibration cable with a tip mass[J]. IEEE Transactions on Automatic Control, 2000, 45(1): 89-93.
- [6] ZHANG Shuang, HE Wei, GE S S. Modeling and control of a nonuniform vibrating string under spatiotemporally varying tension and disturbance[J]. IEEE/ASME Transactions on Mechatronics, 2012, 17(6): 1196-1203.
- [7] HE Wei, ZHANG Shuang, GE S S. Adaptive control of a flexible crane system with the boundary output constraint[J]. IEEE Transactions on Industrial Electronics, 2014, 61(8): 4126-4133.
- [8] HE Wei, GE S S. Cooperative control of a nonuniform gantry crane with constrained tension[J]. Automatica, 2016, 66: 146-154.
- [9] ZHANG Shuang, HE Wei, HUANG Deqing. Active vibration control for a flexible string system with input backlash[J]. IET Control Theory and Applications, 2016, 10(7): 800-805.
- [10] SHI DH, HOU S H, FENG D X. Feedback stabilization of a Timoshenko beam with an end mass[J]. International Journal of Control, 1998, 69(2): 285-300.
- [11] XU Genqi, YUNG S P, LI L K. Stabilization of wave systems with input delay in the boundary control[J]. ESAIM: Control Optimization and Calculus of Variations, 2006, 12(4): 770-785.
- [12] XU Genqi, WANG Hongxia. Stabilization of Timoshenko beam system with delay in the boundary control[J]. International Journal of Control, 2013, 86(6): 1165-1178.
- [13] SHANG Yingfeng, XU Genqi. Stabilization of an Euler-Bernoulli beam with input delay in the boundary control[J]. Systems & Control Letters, 2012, 61(11): 1069-1078.
- [14] WANG Hongxia, XU Genqi. Exponential stabilization of 1-d wave equation with input delay[J]. Wseas Transactions on Mathematics, 2012, 12(10/11/12): 1001-1013.
- [15] KRSTIC M. Delay Compensation for Nonlinear, Adaptive, and PDE Systems[M]. Boston: Birkhäuser Boston Press, 2009.
- [16] BEKIARIS-LIBERIS N, KRSTIC M. Nonlinear Control under Nonconstant Delays[M]. Philadelphia: SIAM, 2013.
- [17] LIU Weijiu. Boundary feedback stabilization of an unstable heat equation[J]. SIAM Journal on Control and Optimization, 2003, 42(3): 1033-1043.
- [18] KRSTIC M, SMYSHLYAEV A. Boundary Control of PDEs: A Course on Backstepping Designs[M]. Philadelphia: SIAM, 2008.
- [19] 郭宝珠, 柴树根. 无穷维线性系统控制理论[M]. 北京: 科学出版社, 2012.
- [20] TUCSNAK M, WEISS G. Observation and Control for Operator Semigroups[M]. Basel: Birkhauser-Verlag, 2009.
- [21] PAZY A. Semigroups of Linear Operators and Applications to Partial Differential Equations[M]. Berlin: Springer-Verlag, 1983.